# Initialization for Ad Hoc Radio Networks with Carrier Sensing and Collision Detection

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5th International Conference on AD-HOC Networks & Wireless 2006

# Outline

# Problem

## Initialization problem

The initialization is a task of assigning to the $n$ stations of the radio network distinct $ID$ numbers in the range 1 to $n$.

# Outline

## Carrier sensing

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3. IEEE 802.11

# Outline

# Introduction
## Previous randomized algorithms

- Nakano-Olariu (2000) : without carrier sensing
  - Known number of stations
    Time: $2.718 \cdot n + O(\sqrt{n \log n})$
  - Unknown number of stations
    Time: $3.333 \cdot n + O(\sqrt{n \log n})$
- Cai-Lu-Wang (2003) : with carrier sensing
  - Known number of stations
  - Unknown number of stations

Probability at least $1 - \frac{1}{n}$. Time complexity of Cai-Lu-Wang algorithms are better than of Nakano-Olariu ones and will be discussed later.

# Outline

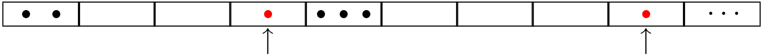
# Nakano - Olariu algorithm

## Known number of stations



### Sketch of algorithm

**for** $m := n$ **downto** 1 **do**
    **repeat**
        each station transmit with probability $\frac{1}{m}$
    **until** channel is SINGLE
    single station sets $id := n - m + 1$ and leaves



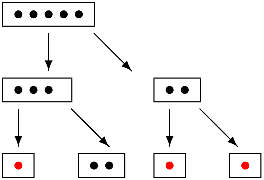
We should play $2.718 \cdot n + O(\sqrt{n \log n})$ times if we want each station to transmit in some slot.

# Nakano - Olariu algorithm

## Unknown number of stations

### Description of the algorithm

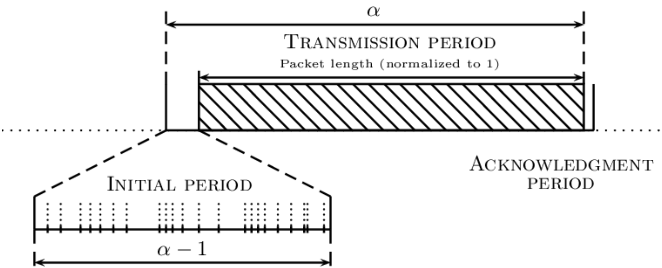
There are $n$ stations. They are divided into groups. If only one station is in the group we called it a **winner**. If not, then each station from the group flips a coin with probability $\frac{1}{2}$ and according to the result goes into a subgroup.



We should play $3.333 \cdot n + O(\sqrt{n \log n})$ times if we want each station to win in some slot (with probability at least $1 - \frac{1}{n}$).

# Outline

## The slot structure



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2. if the channel is **idle** then the station starts transmitting a short signal

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4. go to the next slot with remaining stations

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What is the optimal probability $p^*$?

# Outline

# Known number of stations

## Good configurations

The probability of successful transmission is equal

$$\Pr[(\exists\, 1 \leqslant i \leqslant n)\, (X_{(i)} - X_{(i-1)} > \delta, X_{(i+1)} - X_{(i)} > \delta)]$$

where $X_i$ (for $i \in [1, n]$) is a random time selected by station and $\delta$ is the propagation delay.

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How to calculate the probability?

# Known number of stations
## Combinatorial classes



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$$S(\circ) \times (\bullet \times S_{<D}(\circ))^a \times (\bullet \times S_{\geq D}(\circ))^2 \times (\bullet \times S(\circ))^{n-2-a}$$

Its generating function is $F_a(z) = \frac{(1-z^D)^a z^{2D} z^n}{(1-z)^{n+1}}$. Binomial identities, Stirling numbers, going back to continuous model:

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**Theorem**

$P[success] \approx 2(1-\delta)^n - (1-2\delta)^n$

# Known number of stations

## Adding flexibility

Now: each station transmits in a slot with probability $p = \frac{a}{n}$. Then

$$P[success] \geq 2(1 - \frac{\delta a}{n})^n - (1 - \frac{2\delta a}{n})^n - (1 - \frac{a}{n})^n.$$

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Using Chernoff bound we get

**Theorem**

*If $a \approx \ln(\frac{1}{2\delta^2}) - \ln\ln(\frac{1}{2\delta^2})$ then after $\frac{1}{1-\delta^2} n + O(\sqrt{n \ln n})$ slots each station transmit with probability at least $1 - \frac{1}{n}$.*

# Outline

# Known number of stations

## Comparison

### Comparison

CLW: Cai-Lu-Wang algorithm
CKZ: Cichon-Kutylowski-Zawada algorithm

| $\lambda$ | CLW (2003) | CKZ (2006) |
|---|---|---|
| 0.00001 | $1.0177 \cdot n$ | $1.00088 \cdot n$ |
| 0.0001 | $1.0500 \cdot n$ | $1.00400 \cdot n$ |
| 0.001 | $1.1500 \cdot n$ | $1.01900 \cdot n$ |

Time complexity of the old solution of Nakamo and Olariu is $2.781 \cdot n$

# Outline

# Unknown number of stations
## Sketch of algorithm

Fix probability $p$.

**Basic idea**

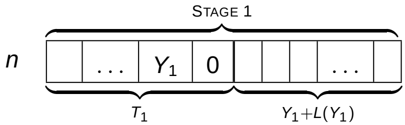
1. all stations flip a coin with probability of success $p$
2. we repeat round (1) until all stations fail
3. stations from **last but one** round we call **winners**, they use the strategy from our previous algorithm
4. go back to (1) with remaining stations

What is the optimal probability $p^*$?

# Sketch of the algorithm

## What we need to calculate






### Main steps

1. the number of winners $Y_1(n)$

# Sketch of the algorithm

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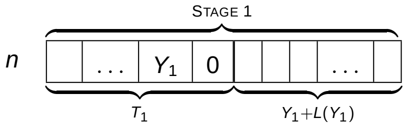


### Main steps

1. the number of winners $Y_1(n)$
2. the number of rounds $T_1(n)$

# Sketch of the algorithm

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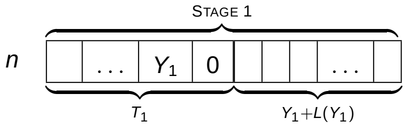


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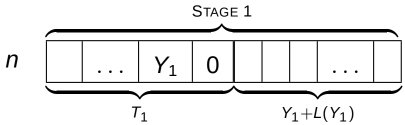


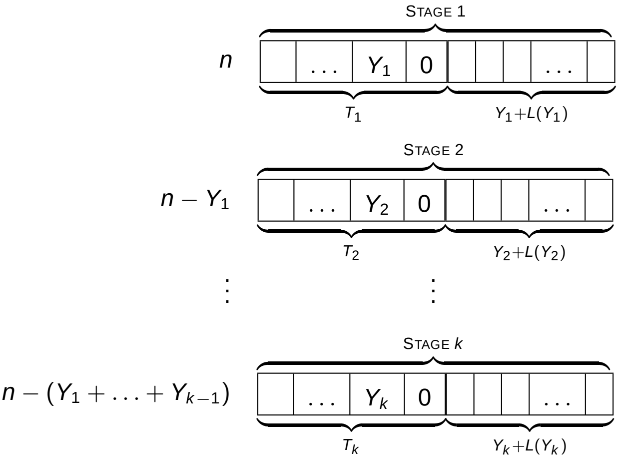

### Main steps

1. the number of winners $Y_1(n)$
2. the number of rounds $T_1(n)$
3. the number of collisions in second part of the stage $L_1(n)$
4. the total number of additional slots $H_1(n) = T_1(n) + L(Y_1(n))$

# Sketch of the algorithm
## What we need to calculate







# Outline

Initialization for Ad Hoc Radio Networks

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Known number of stations
Algorithm
Analysis
Comparison

Unknown number of stations
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**Theorem**

*Let $Y(n)$ be random variable denoting the number of winners, where n is the number of stations. Then*

$$E[Y(n)] = \frac{n(1-p)}{p\ln(1/p)}\left(\frac{1}{n} + 2\sum_{k=1}^{\infty}\Re[B(n, 1+\frac{2k\pi i}{\ln(p)})]\right)$$

*where*

$$B(n,z) = \frac{\Gamma(n)\Gamma(z)}{\Gamma(n+z)}.$$

# Unknown number of stations

## Number of rounds

**Theorem**

*Let $T(n)$ be a random variable denoting the number of rounds such that the number of winners becomes 0, when we start with n stations. Then*

$$\mathbf{E}[T(n)] = \frac{1}{2} + \frac{H_n}{\log(1/p)} + \frac{2}{\log(1/p)} \sum_{k=1}^{\infty} \Re\left[\mathrm{B}\left(n+1, \frac{2k\pi i}{\log(p)}\right)\right]$$

*where $H_n = \sum_{k=1}^{n} \frac{1}{k}$ is the n-th harmonic number.*

# Unknown number of stations

## Number of wasted slots

**Theorem**

*Let $L(Y(n))$ be the number of additional slots in the second part of the stage. Then $E[L(Y(n))]\frac{\ln(1/p)}{n(1-p)}$ equals*

$$\frac{1}{n(p-\delta)} - \frac{1}{pn} + \frac{2}{p-\delta}\sum_{k=1}^{\infty}\Re\left(\left(\frac{1-\delta}{p-\delta}\right)^{\frac{2\pi i}{\ln(p)}} B\left(n, 1+\frac{2k\pi i}{\ln(p)}\right)\right) +$$

$$\frac{2}{p}\sum_{k=1}^{\infty}\Re\left(B\left(n, 1+\frac{2k\pi i}{\ln(p)}\right)\right).$$

# Unknown number of stations

## Upper approximation



Let $H(n) = T(n) + L(Y(n))$. Then

$$C(p, \delta, U) = \max_{m \le U} \frac{1}{E[Y(m)]} \cdot \max_{m \le U} E[H(m)]$$

**Theorem**

*Let U be an upper bound on a number of stations. Then the total number of slots is bounded by*

$$(1 + \mathcal{C}(p, \delta, U)) \cdot n.$$

# Unknown number of stations

## Upper approximation on C



### Theorem

$$C(p,\delta,U) \leq \frac{1}{\psi(p)}\left(W(\delta,p,U) + \frac{1}{2} + \frac{H_U}{\ln(1/p)}\right)$$

*where*

1. $\psi(p) = \frac{1-p}{p\ln(1/p)}\left(1 - 2\sqrt{\frac{2\pi^2}{\ln(1/p)\sinh(2\pi^2/\ln(1/p))}}\right)$,
2. $W(\delta,p,U) = \max_{m\leq U} E[L(Y(m))]$.

# Outline

# Unknown number of stations
## Comparison with simulations

Let $\mathcal{C}(p^*, \delta, U)) = \min_p \mathcal{C}(p, \delta, U)$.

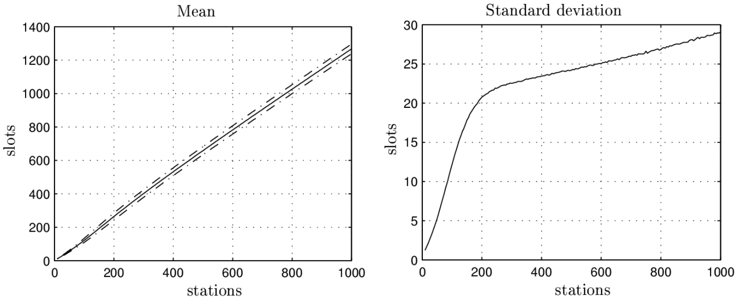
Table: Results for $\delta = 0.001$

| U | $p^*$ | $(1 + \mathcal{C}(p^*, \delta, U)) \cdot n$ | simulations |
|---|---|---|---|
| 100 | 0.037678 | $1.3271 \cdot n$ | $1.3168 \cdot n$ |
| 1000 | 0.0267521 | $1.3998 \cdot n$ | $1.3398 \cdot n$ |
| 10000 | 0.0232507 | $1.4677 \cdot n$ | $1.3482 \cdot n$ |

**Corollary**

Our estimation $\mathcal{C}(p, \delta, U)$ is very precise.

# Simulation results



## Comparison

CLW: Cai-Lu-Wang algorithm
CKZ: Cichon-Kutylowski-Zawada algorithm

Table: Results for $\lambda = 0.005$

| U | $p^*$ | CLW 2003 | CKZ 2006 |
|---|---|---|---|
| 100 | 0.0423848 | $\leq 1.6162 \cdot n$ | $\leq 1.5927 \cdot n$ |
| 1000 | 0.0267521 | $\leq 1.7497 \cdot n$ | $\leq 1.6381 \cdot n$ |
| 10000 | 0.0232507 | $\leq 1.9199 \cdot n$ | $\leq 1.7647 \cdot n$ |

# THANK YOU