# Adaptive Initialization Algorithm for Ad Hoc Radio Networks with Carrier Sensing

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# Known solutions
## History

- Nakano-Olariu (2000)
  - Known number of stations: without delay of signal propagation
    Time: $e \cdot n + O(\sqrt{n \log n})$
  - Unknown number of stations
    Time: $\frac{10}{3} \cdot n + O(\sqrt{n \log n})$
- Cai-Lu-Wang (2003) : with delay of signal propagation
  - Known number of stations
  - Unknown number of stations

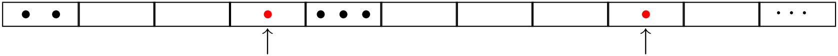
Probability at least $1 - \frac{1}{n}$. Time complexity of Cai-Lu-Wang algorithms are better than of Nakano-Olariu ones - will be discussed later.

# Nakano - Olariu algorithm

## Known number of stations

There are $n$ stations. Time is divided into small slots.

1. Put $k = 0$.
2. Each station tries to transmit with probability $p = \frac{1}{n-k}$. If only one station chooses a given slot then it is a **winner**. Repeat this until there is a winner.
3. Put $k = k + 1$ and goto step 2

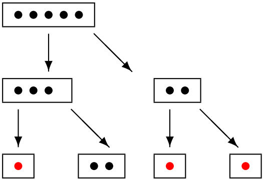


We should play $e \cdot n + O(\sqrt{n \log n})$ times if we want each station to win in some slot.

# Nakana - Olariu algorithm

## Unknown number of stations

There are $n$ stations. They are divided into groups.
If only one station is in the group it is a **winner**. If not, then each station from the group flips a coin with probability $\frac{1}{2}$ and according to the result goes into a subgroup.



We should play $\frac{10}{3} \cdot n + O(\sqrt{n \log n})$ times if we want each station to win in some slot (with probability at least $1 - \frac{1}{n}$).

# Introduction

# Known number of stations

## Sketch of algorithm

Fix probability $p$ and divide time into small slots

### Basic idea

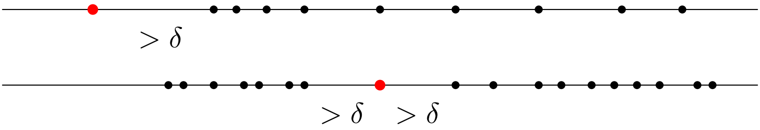
1. in each slot each station with probability $p$ choose a random time $t$
2. if channel is idle then station starts transmission
3. if in time interval $[t, t+\delta]$ there is no collision its *Id* is the slot number and transmit to the end of slot; else stop transmission
4. go to the next slot with remaining stations

What is the optimal probability $p^*$?

# Known number of stations

## Good configurations

The following two situations are good in a slot:




($\delta$ is the normalized delay). How to estimate the probability?
We consider a discretization of this problem:

# Analysis

# Known number of stations

## Combinatorial classes

We use the technology of combinatorial classes:

$$S(\circ) \times (\bullet \times S_{<D}(\circ))^a \times (\bullet \times S_{\geq D}(\circ))^2 \times (\bullet \times S(\circ))^{n-2-a}$$

Its generating function is $F_a(z) = \frac{(1-z^D)^a z^{2D} z^n}{(1-z)^{n+1}}$. Binomial identities, Stirling numbers, going back to continuous model:

**Theorem**

$P[success] \approx 2(1-\delta)^n - (1-2\delta)^n$

# Known number of stations

## Adding flexibility

Now: each station transmits in a slot with probability $p = \frac{a}{n}$.
Then

$$P[success] \geq 2(1 - \frac{\delta a}{n})^n - (1 - \frac{2\delta a}{n})^n - (1 - \frac{a}{n})^n.$$

Using Chernoff bound we get

**Theorem**

*If $a \approx \ln(\frac{1}{2\delta^2}) - \ln\ln(\frac{1}{2\delta^2})$ then after $\frac{1}{1-\delta^2}n + O(\sqrt{n \ln n})$ slots each station transmit with probability at least $1 - \frac{1}{n}$.*

# Known number of stations
## Comparison

### Comparison

CLW: Cai-Lu-Wang algorithm
CKZ: Cichon-Kutylowski-Zawada algorithm

| $\lambda$ | CLW (2003) | CKZ (2006) |
|---|---|---|
| 0.00001 | $1.0177 \cdot n$ | $1.00088 \cdot n$ |
| 0.0001 | $1.0500 \cdot n$ | $1.00400 \cdot n$ |
| 0.001 | $1.1500 \cdot n$ | $1.01900 \cdot n$ |

Time complexity of the old solution of Nakama and Olariu was about $2.781 \cdot n$

# Introduction

# Unknown number of stations

## Sketch of algorithm

Fix probability $p$.

### Basic idea

1. while there are stations without identifiers
2. all stations flip a coin with probability of success $p$
3. we repeat (3) until all are losers
4. all stations from **last but one** stage are **winners**; they use strategy from our previous algorithm (stage 2)
5. go back to (2) with remaining stations

What is the optimal probability $p^*$?

# Unknown number of stations

## What we need to calculate

### Main steps of analysis

1. the number of winners $Y(n)$
2. the length of each main round $T(n)$
3. the number of collisions in second stage $L(n)$
4. the total number of additional slots $H(n) = T(n) + L(Y(n))$

# Unknown number of stations

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# Analysis

# Unknown number of stations

## Number of winners

**Theorem**

*Let $Y(n)$ be the number of winners where $n$ is the number of stations. Then*

$$E[Y(n)] = \frac{n(1-p)}{p\ln(1/p)}\left(\frac{1}{n} + 2\sum_{k=1}^{\infty}\Re[B(n, 1 + \frac{2k\pi i}{\ln(p)})]\right)$$

*where*

$$B(n,z) = \frac{\Gamma(n)\Gamma(z)}{\Gamma(n+z)}.$$

# Unknown number of stations
## Example of proof

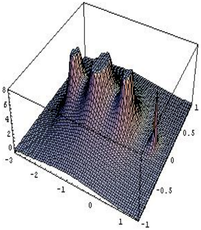
**Proof.**

1. show that $E[Y(n)] = \frac{n(1-p)}{p\ln(1/p)} \sum_{k=0}^{n-1} \binom{n-1}{k} (-1)^k \frac{1}{1-p^{k-1}}$;
2. define function $f(z) = \frac{1}{1-p^{-z+1}}$;
3. show that $E[Y(n)] = \sum_{k=0}^{n} Res[B(n,z)f(z)|z=-k]$;
4. use Cauchy Theorem and do some reductions to finish the proof;

□

$Res[g(z)|z=a]$ denotes the residuum of $g(z)$ at the point $a$.

# Unknown number of stations
## Residues




z = |B(2,z) f(z)|

### Additional useful calculations

Let $z_k = 1 + \frac{2k\pi i}{\ln(p)}$. In further calculations we need to know $Res[B(n,z)f(z)|z = z_k]$

1. $Res[B(n,z)f(z)|z = z_0] = \frac{1}{n\ln(p)}$
2. $Res[B(n,z)f(z)|z = z_k] = \frac{\Gamma(n)\Gamma(z_k)}{\Gamma(n+z_k)\ln(p)}$ if $k \neq 0$

# Unknown number of stations

## Number of rounds

### Theorem

*Let $T(n)$ be a random variable denoting the number of rounds such that the number of winners becomes 0, when we start with n winners. Then*

$$\mathbf{E}[T(n)] = \frac{1}{2} + \frac{H_n}{\log(1/p)} + \frac{2}{\log(1/p)} \sum_{k=1}^{\infty} \Re\left[\mathrm{B}\left(n+1, \frac{2k\pi i}{\log(p)}\right)\right]$$

*where $H_n = \sum_{k=1}^{n} \frac{1}{k}$ is the n-th harmonic number.*

# Unknown number of stations

## Number of wasted slots

### Theorem

*Let $E[L(Y(n))]$ be the number of slots in the second round where two stations transmit. Then $E[L(Y(n))]\frac{\ln(1/p)}{n(1-p)}$ equals*

$$\frac{1}{n(p-\delta)} - \frac{1}{pn} + \frac{2}{p-\delta}\sum_{k=1}^{\infty}\Re\left(\left(\frac{1-\delta}{p-\delta}\right)^{\frac{2\pi i}{\ln(p)}} B\left(n, 1+\frac{2k\pi i}{\ln(p)}\right)\right) +$$

$$\frac{2}{p}\sum_{k=1}^{\infty}\Re\left(B\left(n, 1+\frac{2k\pi i}{\ln(p)}\right)\right).$$

# Unknown number of stations
## Upper approximation

Let $H(n) = T(n) + L(Y(n))$, $Z(n) = n - Y(n)$. Let

$$C(p,\delta,U) = \min_{m \leq U} \frac{1}{E[Y(m)]} \cdot \min_{m \leq U} \sum_{r=0}^{m} P[Z(m) = r] \cdot E[H(r)]$$

**Theorem**

*Let $U$ be an upper bound on a number of slots. Then the total number of slots is bounded by*

$$(1 + \mathcal{C}(p,\delta,U)) \cdot n.$$

# Unknown number of stations

## Upper approximation on C

### Theorem

$$C(p,\delta,U) \leq \frac{1}{\psi(p)}\left(W(\delta,p,U) + \frac{1}{2} + \frac{H_U}{\ln(1/p)}\right)$$

*where*

1. $\psi(p) = \frac{1-p}{p\ln(1/p)}\left(1 - 2\sqrt{\frac{2\pi^2}{\ln(1/p)\sinh(2\pi^2/\ln(1/p))}}\right)$,
2. $W(\delta,p,U) = \max_{m\leq U} E[L(Y(m))]$.

# Conclusions

# Unknown number of stations

## Comparison with simulations

Let $\mathcal{C}(p^*, \delta, U)) = \min_p \mathcal{C}(p, \delta, U)$.

Table: Results for $\delta = 0.001$

| U | $p^*$ | $(1 + \mathcal{C}(p^*, \delta, U)) \cdot n$ | simulations |
|---|---|---|---|
| 100 | 0.037678 | $1.3271 \cdot n$ | $1.3168 \cdot n$ |
| 1000 | 0.0267521 | $1.3998 \cdot n$ | $1.3398 \cdot n$ |
| 10000 | 0.0232507 | $1.4677 \cdot n$ | $1.3482 \cdot n$ |

**Corollary**

Our estimation $\mathcal{C}(p, \delta, U)$ is very precise.

# CKZ solution

## Capmarison with Cai-Lu-Wang algorithm

### Comparison

CLW: Cai-Lu-Wang algorithm
CKZ: Cichon-Kutylowski-Zawada algorithm

Table: Results for $\lambda = 0.005$

| U | $p^*$ | CLW (2003) | CKZ 2006 |
|---|---|---|---|
| 100 | 0.0423848 | $\leq 1.6162 \cdot n$ | $\leq 1.5927 \cdot n$ |
| 1000 | 0.0267521 | $\leq 1.7497 \cdot n$ | $\leq 1.6381 \cdot n$ |
| 10000 | 0.0232507 | $\leq 1.9199 \cdot n$ | $\leq 1.7647 \cdot n$ |

# Future works

## Changing the probability

### Non-uniform probability

We calculated another way of generating the random time when a station tries to start transmission: the density (on $[0, 1]$) was

$$\varphi(x) = 2x.$$

Then we obtained slightly better results than in the above algorithm.

# Future works
## Example: new technology required

Look at distribution of probabilities in $n$ slots.

| $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ | $p_7$ | $\cdots$ | $p_n$ |
|---|---|---|---|---|---|---|---|---|

Let $N(p, n, s) = E$[number of winners] in ,,Nakano-Olariu game".

### Optimization of Nakano - Olariu algorithm

- goal: $\max_p N(p, n, s)$
- constraints: $\bigwedge_{i=1}^{n}(0 \leq p_i \leq 1)$

A similar optimization problems should be solved for more flexible versions of Cai-Lu-Wang algorithms with carrier sensing.

# Future works

## Possible future results

### Possible applications

1. simpler algorithms
2. low energy algorithms
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