Key Levels and Securing Key Predistribution Against Node Captures

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ACNS 2009

Application Scenario

simple devices, symmetric methods, ad hoc connections

Network

- a network of simple devices, equipped with symmetric algorithms only
- unpredictable which devices will communicate
- all devices from the same provider

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Requirement

- no plaintext transmission
- two devices establish a session key when they meet

Random Key Predistribution

simple devices, symmetric methods

Initialization

- The system provider keeps a secret pool $\mathcal{K}$ of keys selected at random.
- Before being used a device receives k keys from $\mathcal{K}$ chosen at random.

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Setting up a connection between A and B

- A and B determine the keys they share, say $k_{i_1}, \ldots, k_{i_t}$,
- A and B compute the session key

$$\mathcal{K} = F(k_{i_1}, \ldots, k_{i_t}, A, B, ...)$$

based on the birthday paradox

Attack
node captures

Collecting keys

An adversary

1. gets devices
2. retrieves the keys contained inside (may be in a destructive way)

Scale of the problem

- no physical protection of the devices
- cheap devices are not tamper proof

Goal

Improve the situation!

- many diverse proposals in the literature,
- **we provide an additional security mechanism for almost all predstribution techniques**

Key Levels Technique

T Levels Scheme

1. each single key k from the basic method corresponds to an set of keys

$$K_1, K_2, \ldots, K_T$$

2. the keys related in a one-way fashion:

$$K_1 = K \quad \text{and} \quad K_{i+1} = G(K_i) \quad \text{for } i = 1, \ldots, T-1$$

where G is easy to compute but infeasible to invert

Establishing a Connection

T level scheme

Mechanism

if A holds K_i and B holds K_j, then $K_{\max(i,j)}$ used for establishing the shared key

computing K_s from K_t, for $s > t$, is easy,

it is infeasible for $s < t$

Gain

if an adversary holds

$$K_t \quad \text{for } t > \max(i,j),$$

then the connection between A and B is secure

(with A, B and the adversary holding (a version of) K)

Problems

How to assign the levels

1. the uniform distribution is not optimal
2. example: the optimal pbb of choosing K_1, K_2, K_3, K_4:

$$0.437055, \quad 0.218527, \quad 0.182106, \quad 0.162312$$

3. we show an effective procedure to find the optimal probabilities

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Probability of adversary's failure

assumption: A, B and the adversary use a version of K

1. for 2 levels it is $\frac{4}{27}$, pbb increases with the number of levels
2. for infinite number of levels:
   - it reaches $\frac{1}{3}$
   - no matter what probability density is used

Attack Cost

the expected number of devices corrupted until a connection becomes insecure

Theorem (2 level case, p is the probability to choose level 1)

Let $L_{m,p}$ denote the number of steps after which adversary collects all keys for compromising connection based on m shared keys. Then

$$E[L_{m,p}] = \int_0^\infty \left(1 - \frac{H(t)}{e^t}\right) dt \,, \tag{1}$$

where $H(z) = \left(e^{z/m} - 1 - p^2(e^{qz/m} - 1)\right)^m$ and $q = 1 - p$.

Attack Cost

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Corollary

- For $m = 1$ the optimal value of p is 0.5; then $E[L_m] \approx 1.25$.
- If $m = 10$, then the optimal value of p is 0.32164; in this case we get $E[L_m] = 40.9724$, so $E[L_m] = 1.39887 \cdot m \cdot H_m$, where $H_m =$ the mth harmonic number. So the actual cost of breaking the transmission is increased by $\approx 40\%$

Very large number of levels

From factor 1 improve to 1.5 as a limit value.

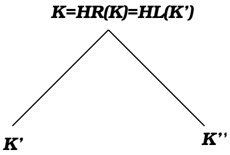
Trees

an extension with no *weak keys*

Idea

Instead of a single key K or a chain of keys $K_0, K_1 \ldots$, we can construct the following tree $T_{\hat{K}}$ of keys:

- each node of the tree is labeled with a key, the root is labeled with $\hat{K}$,
- if a node is labeled with key K, then its parent is labeled with $H_i(K)$, where $i = L, R$



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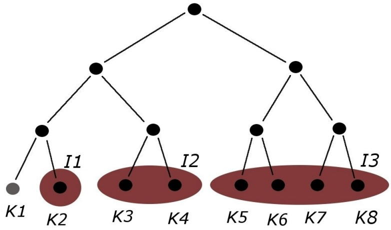
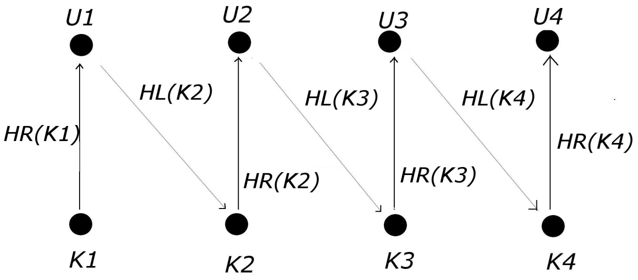


a tree containing keys $K_1, \ldots K_8$, if adversary is holding the key K_1, then the communication between A and B is not broken if they both hold keys from $I1 = \{K_2\}$ or from $I2 = \{K_3, K_4\}$ or from $I3 = \{K_5, K_6, K_7, K_8\}$

Reducing the number of keys in a device

keeping connectivity



1. special choice of keys in the pool
2. the devices do not have to share a key, subsequent keys can be used as well

Conclusions

Further constructions and details

to be presented during ALGOSENSORS'2009

Main features

attack resilience improved moderately, but practically with no cost