Rapid Mixing and Security of Chaum's Electronic Voting

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How do we spend next 18 minutes (more or less) ?

- electronic voting
- David Chaum's voting scheme
- Randomized Partial Checking – method of Markus Jakobsson Ari Jules and Ronald R. Rivest
- our result that relates to Chaum's scheme

Electronic Voting

What is expected ?

- anonymity of voters
- verifiability
- no vote selling
- low cost and efficiency

Chaum's Electronic Voting Scheme - user's point of view

Combines Visual Cryptography and digital processing

- At polling place voter gets "receipts"
- Votes are processed
- At the end of the protocol voter can easy check if the vote is included in the final tally.

Chaum's Electronic Voting Scheme

It has many advantages:

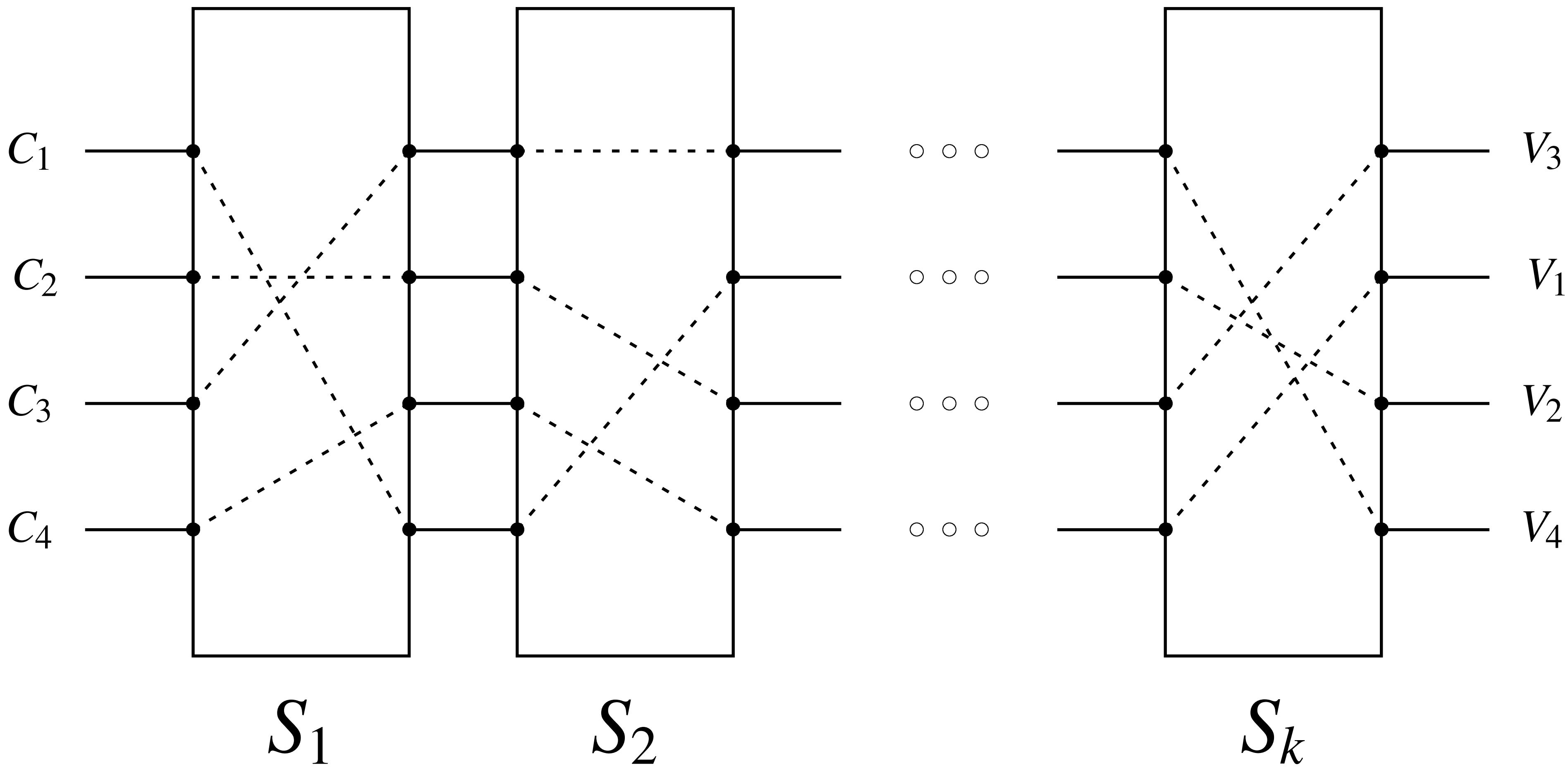
- fairly practical
- verifiable
- "receipts" for voters - no trust in electronic devices required
- no selling votes
- ...it seems that voter obtain anonymity, too

Our aim: Rigid mathematical proof of anonymity.

Electronic processing

- Based on onion-routing
- k MIX-servers $S_1, S_2, ..., S_k$ with encryption and decryption algorithms E_i,D_i
- V-vote is encoded as $C = E_1(E_2(\ldots E_k(V)\ldots))$
- ...and decoded as $V = E_k(E_{k-1}(\ldots E_1(V)\ldots))$
- Each MIX-server peels off one encryption layer and mixes randomly all ballots.

Full anonymity but no verifiability.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C_2
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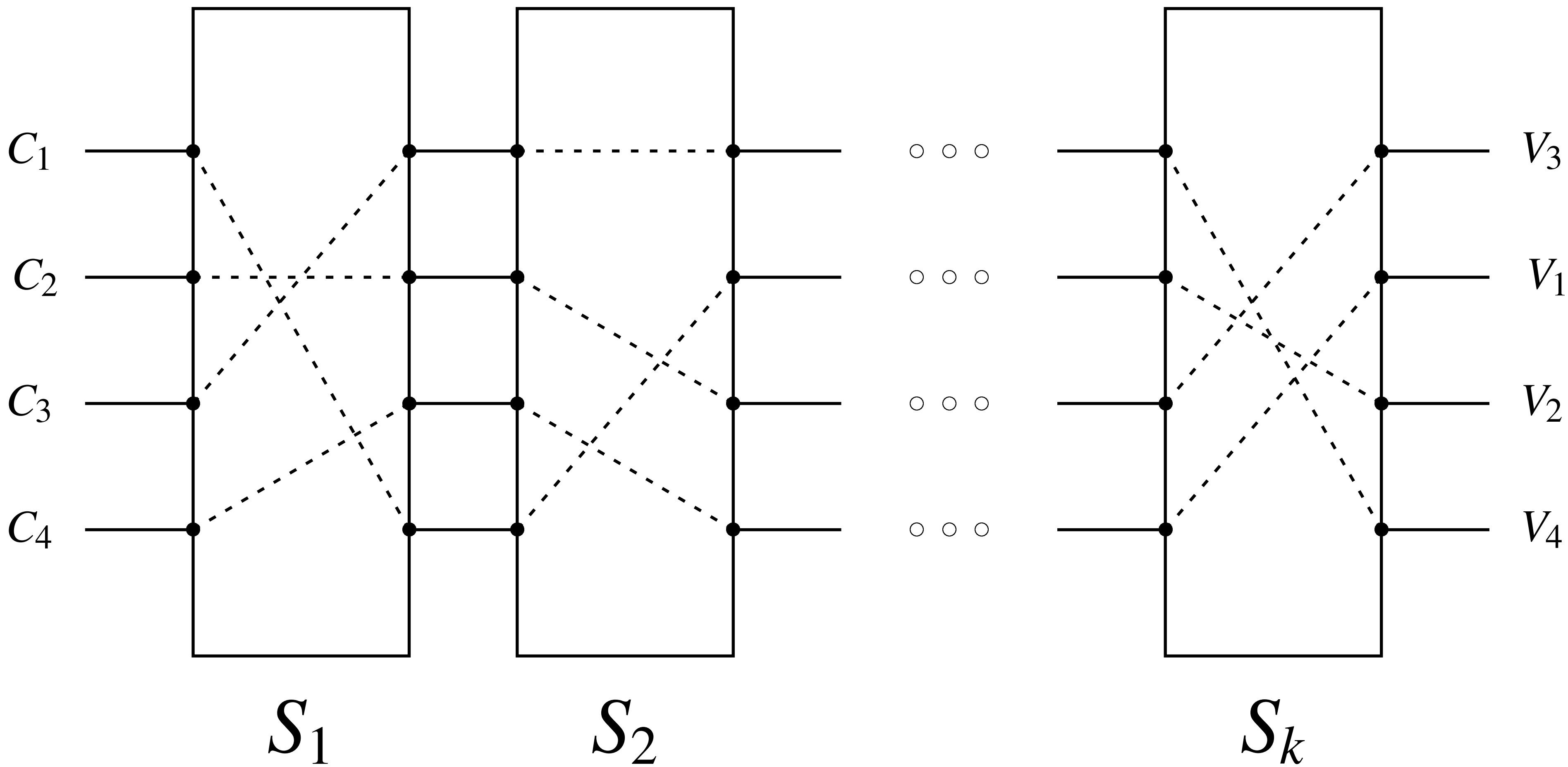
S_1
S_2
S_k

Randomized Partial Checking

- M.Jakobsson, A.Juels, R.R.Rivest "Making Mix Nets Robust for Electronic Voting by Randomized Partial Checking" http://www.rsasecurity.com/rsalabs/staff/bios/mjakobsson/rpcmix/rpcmix.pdf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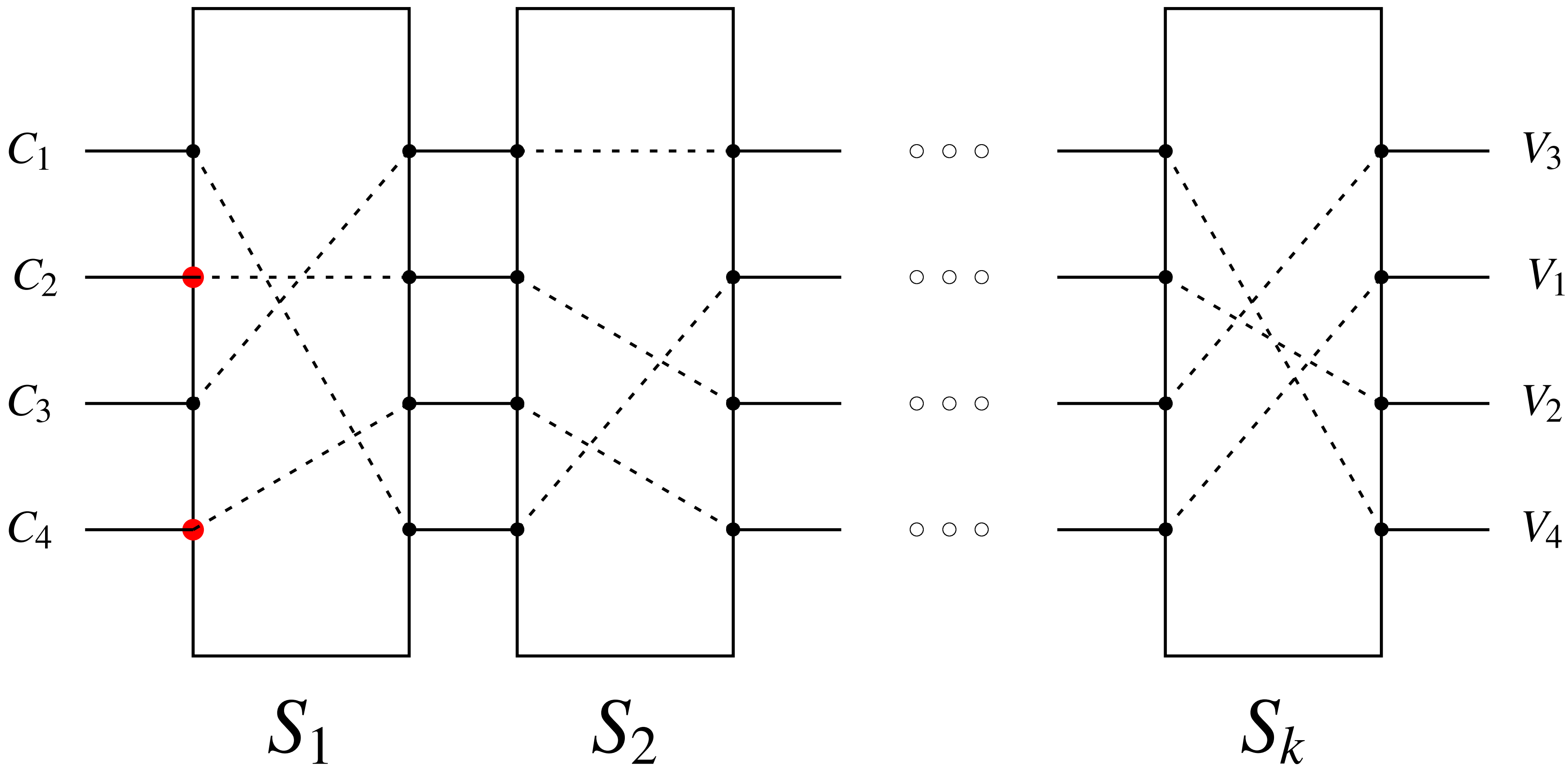
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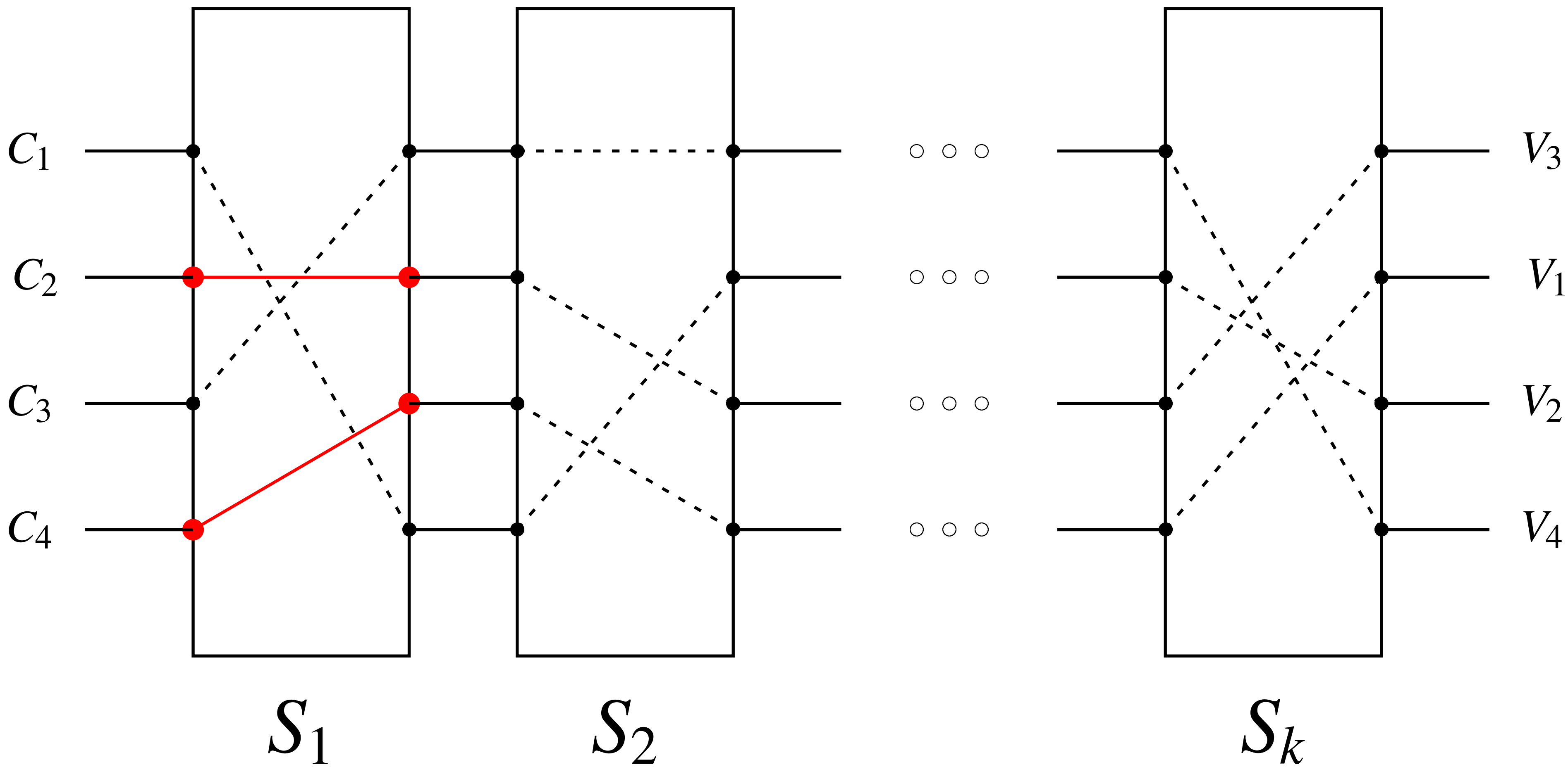
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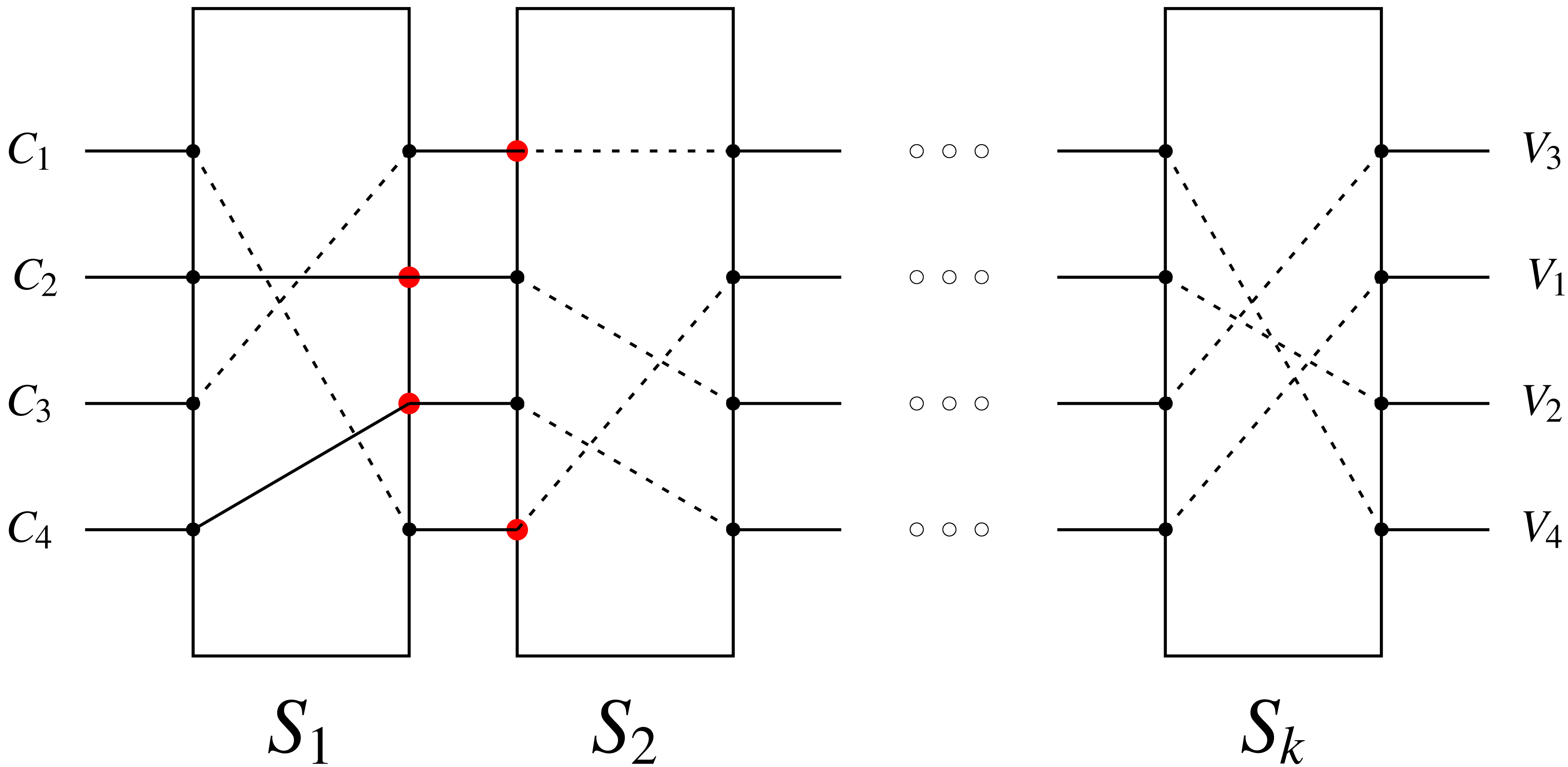
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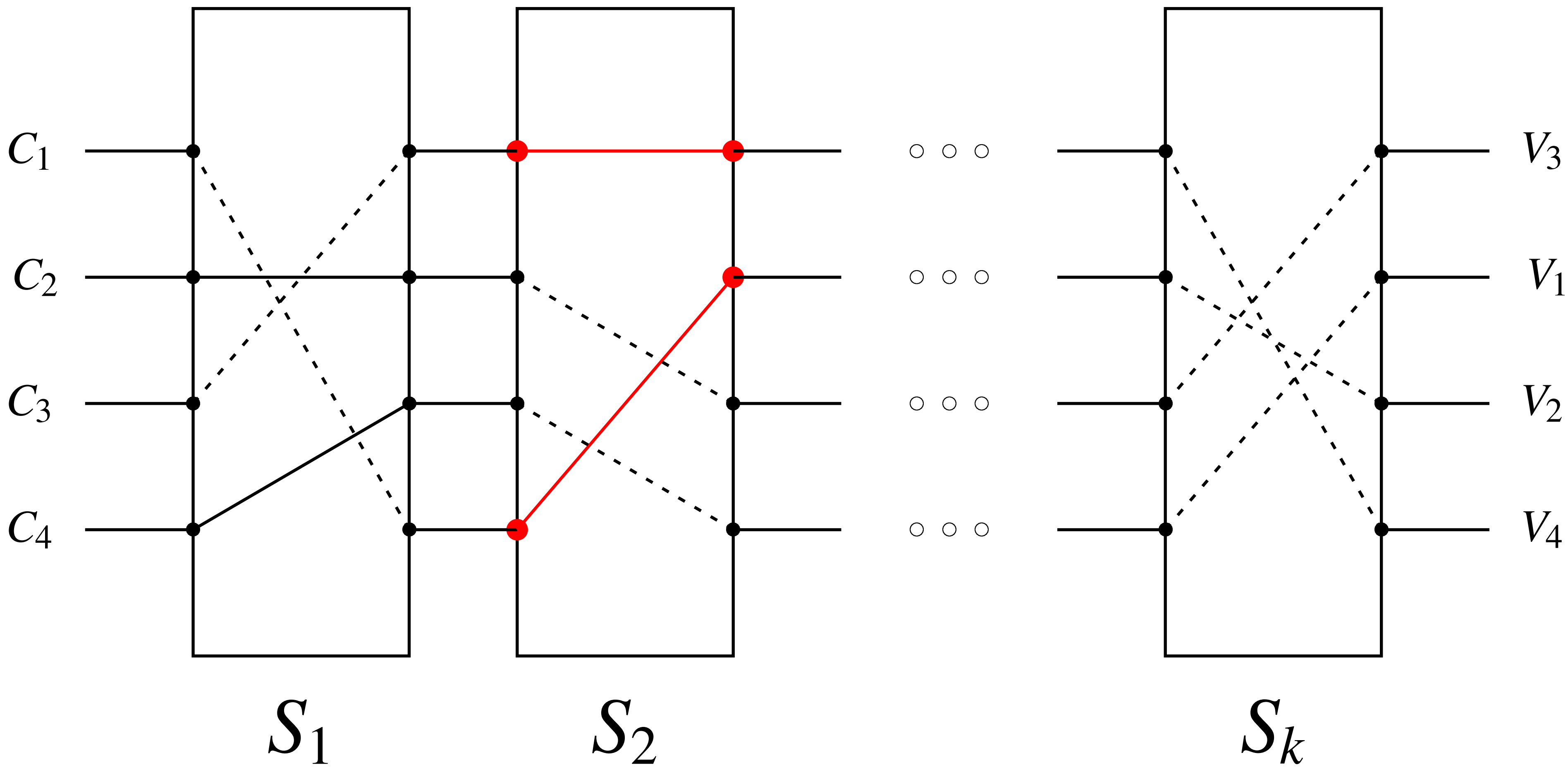
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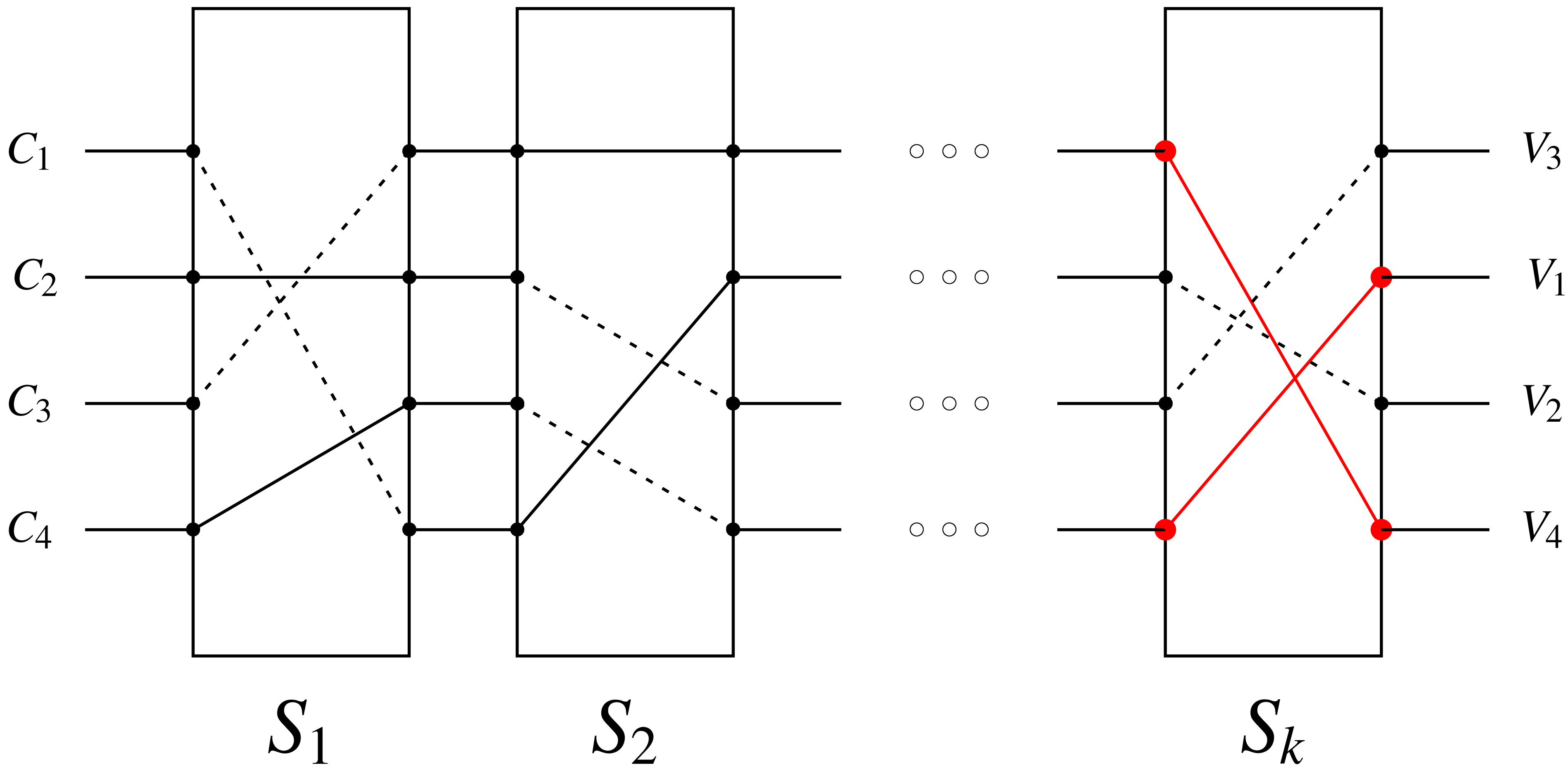
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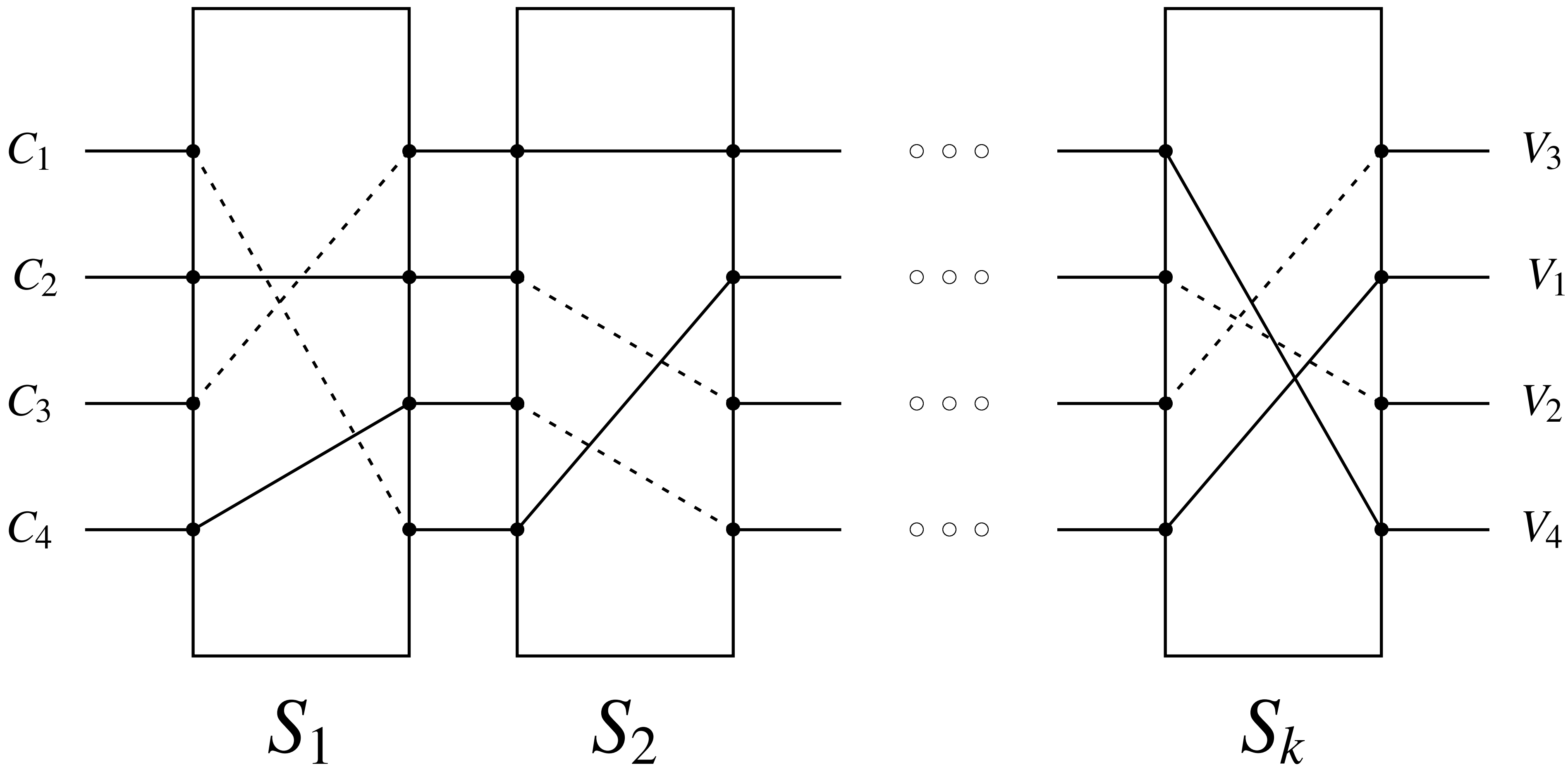
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What is anonymity ?

- Voters are perfectly anonymous iff from the point of view of a passive adversary: **Each permutation between plaintexts at the end and ciphertexts at the beginning of MIX-cascade is equally probable.**
- Π_i - random variable that represents permutation of votes leaving the i–th MIX-server. For perfect anonymity we want Π_k to have the uniform distribution.

Total Variation Distance

For μ_1 and μ_2 over a finite space Ω *total variation distance* defines distance between μ_1 and μ_2.

$$\|\mu_1 - \mu_2\| = \frac{1}{2} \sum_{y \in \Omega} |\mu_1(y) - \mu_2(y)| \ .$$

Main Result

Let n be a number o voters, then

There exists $T = O(1)$ such that the variation distance between distribution of Π_T and the uniform distribution is $O\left(\frac{1}{n}\right)$.

Technicalities

- We treat $\{\Pi_i\}_{i \in \mathbb{N}}$ as a Markov chain.
- This process converges to uniform distributions.
- We need to estimate the rate of convergence.

Coupling

- $\mathcal{L}(\Pi_t)$ - probability distribution of Π_t
- μ -uniform distribution over $\mathbb{S}_n$
- *Coupling* is a stochastic process $(\Pi_t, \Pi_t^\star)$ on the space $\mathbb{S}_n \times \mathbb{S}_n$ such that $\mathcal{L}(\Pi_t^\star)$=$\mathcal{L}(\Pi_t)$
- Coupling Lemma (e.g. in D. Aldous "Random Walks of Finite Groups and Rapidly Mixing" 1983)

$$\|\mathcal{L}(\Pi_t) - \mu\| \leq \mathbf{Pr}[\Pi_t \neq \Pi_t^\star]$$

- We obtain estimation of converging rate by constructing proper coupling.

Path Coupling Method

- extension of well-known *Coupling*
- invented by Russ Bubley and Martin Dyer ("Path Coupling: A Technique for Proving Rapid Mixing in Markov Chains")
- Consist in building copies of stochastic process that differs slightly.
- We consider coupling only for a particular subset of $\mathbb{S}_n \times \mathbb{S}_n$
- Standard tools give $T = O(log(n))$ bound.

Grouping MIX-servers into pairs is important

If halves were chosen independently for all MIX-servers we would obtain $O(log(n))$ only.

Conclusions

- Using Chaum's Scheme we obtain **provable** anonymity
- We can use **constant** number of MIX-servers. Security does not depend on number of voters.
- We can divide mix-cascade into batches of two without loosing anonymity.