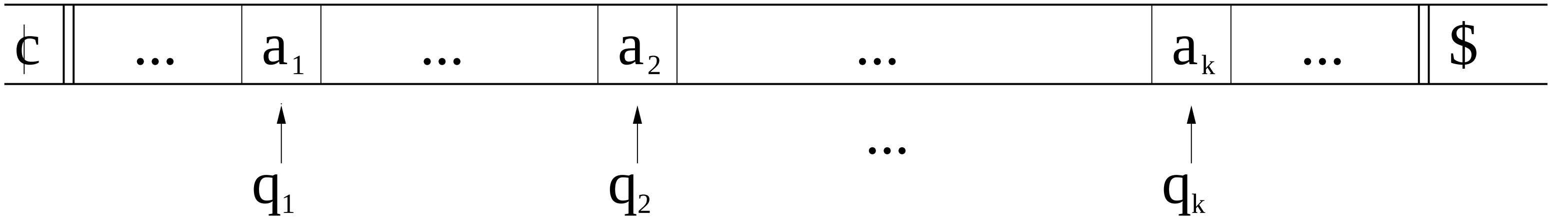
Communication Complexity for Asynchronous Systems of Finite Devices

Tomasz Jurdziński	Uniwersytet Wrocławski, Poland, Technische Universität Chemnitz, Germany
Mirosław Kutyłowski	Uniwersytet Poznański, Poland, Politechnika Wrocławska, Poland
Jan Zatopiański (speaking)	Uniwersytet Wrocławski,Poland

Motivations

- Communication Complexity with limited resources;
- Synchronous and asynchronous distributed computations;
- Capability of asynchronous computations.

Systems of finite automata
- model of distributed systems



- Computations of a constant number of independent finite two-way automata;
- Automata work on a shared, read-only input tape;
- Cooperation: during transitions automata can send messages;

Model

- Alphabet of messages, Δ, with symbol $\perp$ meaning no message;
- Buffers have finite size (for each pair of automata);
- A transition of an automaton depends on input symbols and messages received;
- communication: changes the state, deletes the oldest messages, sends new ones and moves the head:

$$\delta_i : Q_i \times (\Delta \cup \perp)^{k-1} \times \Sigma \longrightarrow Q_i \times (\Delta \cup \perp)^{k-1} \times \{L, R, \perp\}.$$

where k is the number of automata.

Synchronous and asynchronous computations

Complexity measure: the number of messages sent by all automata during computation of the system.

Synchronous Systems: central clock; for all automata transitions are done simultaneously;

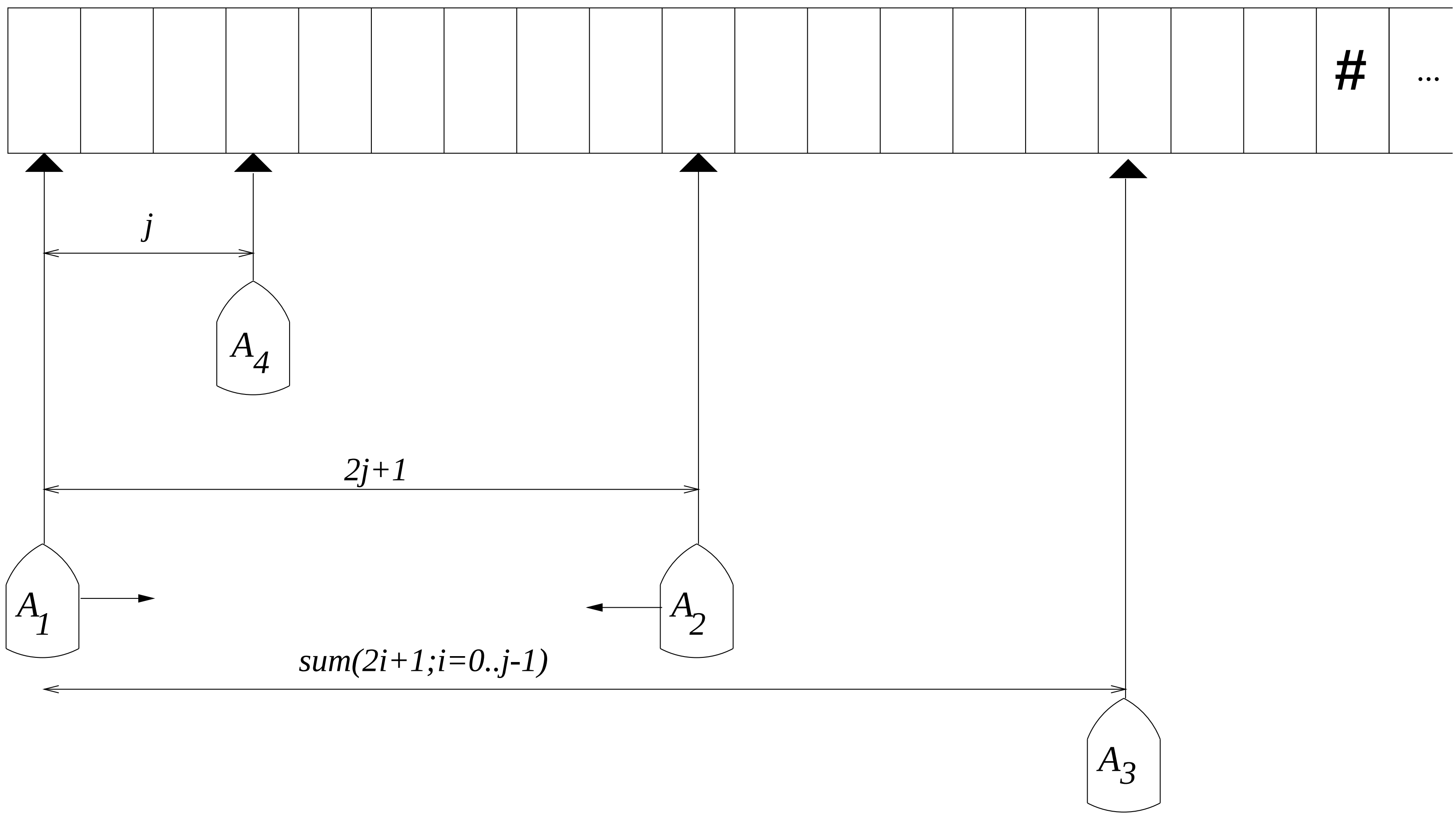
Asynchronous Systems: no common clock; automata work independently, but for each system run, the result must be the same.

Examples

Message complexity:

language	synchronous sys.	asynchronous sys.
$\{a^n b^n\}$	$O(1)$	$\Omega(n)$
$\{w : \|w\|_0 = \|w\|_1\}$	$O(1)$	$\Omega(n)$
$\{w : \sqrt{\|w\|} \in \mathbb{N}\}$	$O(n^{1/2})$	$\Omega(n)$

Counting square root of word length



Problems

- Impact of asynchronism on computational power.

Results 1

1. Asynchronous: $O(1)$ messages is enough for recognizing regular sets, only!
2. there are no languages with asynchronous communication complexity $o(n)\setminus\Omega(1)$.

Proof by analyzing **simple computations:**

At each moment, exactly one automaton (with the least possible number) makes progress. It is one of many possible asynchronous computations.

Results 2

Even for over-linear communication, asynchronous systems are weaker than synchronous ones.

Complexity of language L_{trans}:

	synchronous	asynchronous
upper bound	$O(n)$	$O(n^{3/2})$
lower bound	$\Omega(n)$	$\Omega(n^{3/2}/\log^2(n))$

Lower bound proof by Kolmogorov complexity and conversion to several protocols of the classical two-party communication complexity. communicational protocols.

Results 2

Kolmogorov complexity

complexity of a word - size of a minimal encoding of program computing this word.

$K(x|y) = min\{|p| : p \in \{0,1\}^* \;\&\; T(p,y) = x\}$

Fact: For each n an overwhelming majority of words of length n are “random”.

Definition: The word x is called random” if $K(x) \geq |x| - c\log(|x|)$.

The language L_{trans}

Matrix:

$$U = \begin{bmatrix} u_{1,1} & u_{1,2} & \dots & u_{1,N} \\ u_{2,1} & u_{2,2} & & \\ \vdots & & & \vdots \\ u_{N,1} & & \dots & u_{N,N} \end{bmatrix}$$

and its encoding:

$$\boxed{u_{1,1} \;\dots\; u_{1,N}} \cdots \boxed{u_{N,1} \;\dots\; u_{N,N}} \;\#\; \boxed{u_{1,1} \;\dots\; u_{N,1}} \cdots \boxed{u_{1,N} \;\dots\; u_{N,N}}$$

Upper bounds for L_{trans}

Lemma: Recognizing L_{trans} is possible with $O(n)$ messages on synchronous systems, and with $O(n^{3/2})$ on asynchronous systems.

Sketch of the proof

1. computing the $r = \sqrt{|w_1|}$ for input $w = w_1 \# w_2$ with $O(|w|^{1/2})$ messages;
2. storing the r by the distance between automata;
3. comparing rows of matrix encoded in w_1 to columns from w_2.

Finite automata on random words 1

Property: Finite automata on “long” random word reach states “fast” or “never”.

Finite automata on random words 2

Lemma:

Let M be a two-way DFA, $x \in \Sigma^n$, $|\Sigma| = s$; $K_s(x) > n - c \log_s n$ Let $\mathcal{C}$ be a configuration of M starting in the middle of x, and M does not loop in x. If M starts computation in state $\mathcal{C}$ in the middle of the word x, then M reaches the state q_m after at most $c' \log n$ steps, or does not reach the state q_m until leaving x and scanning some symbols not in Σ.

input: random word x,i.e. $K_s(x) > n - c\log_s n$

$\Downarrow$

Assumption: M cannot reach a state q_{mes} fast
but still can do it outside the word

$\Downarrow$

behavior of M gives the way
to compress the word x

$\Downarrow$

contradiction

Compression of the word

for long enough x there exist sequences

$\{x_{i,\bullet}\}$, $\{x'_{i,\bullet}\}$, $y_{i+1} = x'_{i,L}\ y_i\ x'_{i,R}$

where $x_{i,\bullet}$ are the shortest words such that

M reaches q_m without going outside

$x_{i,L}\ y_i\ x_{i,R}$ and M does not reach q_m

without going outside $x'_{i,L}\ y_i\ x'_{i,R}$

$\Downarrow$

word x cannot contain y_i

Compression of the word

$\Downarrow$

compression by giving program to compute y_i
description of M, C and contents of x without x_{i_0},
and index of x_{i_0} (generated as y_i)

$\Downarrow$

$$K(x) < n - c \log_s n$$

Communication protocols

1. Let S be an asynchronous system recognizing L_{trans};
2. In ith protocol A knows matrix $U/(row(i) \leftarrow x)$, and B knows $U^T = V/(col(i) \leftarrow y)$; x and y are random words;
3. Parties test equivalence $x = y$ by simulating S;

Communication protocols

4. Preprocessing: A and B exchange set of transitions for words x and y;
5. A simulates S when possible;
6. When simulation is not possible - parties exchange information about states of automata sending messages called “important”.

Lower bound for the language L_{trans}

Lemma: Asynchronous systems need $\Omega(n^{3/2}/\log(n))$ messages to recognize a word to L_{trans}.

Proof:

1. Set of protocols $P_1, \ldots, P_N$. Protocols are relative to simple computations;
2. Each protocol needs $\Omega(N/\log(N))$ important messages;
3. Each message can be related to at most k protocols, where k is number of automata in the system;

Lower bound for the language L_{trans}

Lemma: Asynchronous systems need $\Omega(n^{3/2}/\log(n))$ messages to recognize a word to L_{trans}.
Proof(cd):

4. In block of consecutive k^2 important messages there exists at least $\Omega(N/\log(N))$ auxiliary messages;
5. Hence, $\Omega(\frac{(N \cdot N/\log(N))/k}{k^2} \cdot N/\log(N))$ messages must be used.

Open problems

1. Different ways of getting results (without correctness guarantee for any computation)
2. limited asynchronism.
   Partial results for multi-speed systems known.

Bibligraphy1

BTY P. Beame, M. Tompa, and P. , Yan. Communication-space tradeoffs for unrestricted protocols. *SIAM Journal on Computing*, pages 652–661, 1994.

BC A. Borodin and S. Cook. A time-space tradeoff for sorting on a general sequential model of computation. *SIAM Journal on Computing*, pages 287–297, 1982.

CDR Cook, Dwork, and Reischuk. Upper and lower time bounds for parallel random access machines without simultaneous writes. *SIAM Journal on Computing*, pages 87–97, 1986.

Bibligraphy2

STOC93 T. Jiang and M. Li. k one-way heads cannot do string-matching. In *ACM Symposium on Theory of Computing*, pages 62–70. ACM Press, 1993.

DMS M. Dietzfelbinger, W. Maass, and G. Schnitger. The complexity of matrix transposition on one-tape off-line Turing machines. *Theoretical Computer Science*, pages 113–129, 1991.

tju-phd T. Jurdziński. *Communication Aspects of Computation of Systems of Finite Automata*. PhD thesis, Institute of Computer Science, University of Wrocław, Wrocław, 2000.

Bibligraphy3

Cocoon99 T. Jurdziński, K. Kutyłowski, and K. Loryś. Multi-party finite computations. In *Computing and Combinatorics, 5th Annual International Conference, COCOON '99, Tokyo, Japan*, volume 1627 of *Lecture Notes in Computer Science*, pages 318–329. Springer-Verlag, 1999.

dioph T. Jurdziński and M. Kutyłowski. Communication gap for finite memory devices. accepted for ICALP 2002

Nisan E. Kushilevitz and N. Nisan. *Communication Complexity*. Cambridge University Press, 1997.

Bibligraphy4

LTT T. Lam, P. Tiwari, and M. Tompa. Trade-offs between communication and space. *Journal of Computer and System Sciences*, pages 296–315, 1992.

Kolmogorov M. Li and P. Vitanyi. *An Introduction to Kolmogorov Complexity and its Applications*. Springer-Verlag, 1993.