Energy-Efficient Size Approximation of Radio Networks with no Collision Detection

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Radio network model

- a network consists of *stations* (pocket devices)
- communication between stations via a shared radio channel
- common clock
- messages sent in time slots common to all stations

Applications

- sensor networks
- battlefield and rescue operations
- new application areas ...

Advantages

- no routing, no latency
- flexibility - stations may join and leave the network
- no central control vulnerable to an attack
- self-organizing

Disadvantages

- unknown which stations are *off* and *on*
  stations that are on = *alive stations*
- **the number of alive stations is unknown**
- often the stations are indistinguishable
- collisions in communication
  – at least two stations sending at the same time
  $\Rightarrow$ scrambling
- collision indistinguishable from noise

Complexity measures

time - the total number of time slots used

energy -

- a station that *listens* or *sends* in a time slot is *active*, otherwise inactive
- being active causes the main usage of energy
- energy cost of a station = the number of time slots in which it is active
- energy cost of an execution = the maximal energy cost over all stations

Computation Scenarios

the number of active stations is

scenario 1: known

scenario 2: known up to a constant factor

scenario 3: unknown

Example: *Ethernet solution* for leader election

given n alive stations,
at the end exactly one station should have status *leader*

Algorithm repeat until success:

1. each station with probability $\frac{1}{n}$ sends a message and listens
2. if no collision, then the station that has sent is the leader

Example: Ethernet solution for leader election

Properties:

- probability of electing a leader in one trial $\approx \frac{1}{e}$
- within $O(\log n)$ trials a leader should be elected whp
- energy cost $O(\log n)$, equal to time
- **but:** n has to be known in advance

Example: Network initialization

given $\theta(n)$ alive stations,
at the end the station should have unique identifiers $1, 2, \ldots$

Algorithm Nakano, Olariu
Energy Efficient Initialization Protocols for Radio Networks with no Collision Detection. ICPP'2000,
time $O(n)$, energy $O(\log\log n)$

works if the number of alive stations known up to a constant factor

Main Result

Theorem 1 *There is a randomized algorithm for weak no-CD RN such that*

- *with probability* $\geq 1 - \frac{1}{n}$ *a number* n_0 *is found such that*

$$\frac{1}{c} n_0 \leq n \leq c n_0.$$

- *execution time is* $O(\log^{2+\varepsilon} n)$
- *energy cost is* $O((\log \log n)^{\varepsilon})$ *(for any constant* $\varepsilon > 0$*).*

Corollary

Theorem 2 *There is an initialization protocol for a no-CD RN with n stations that*

- *works in scenario III*
- *has time complexity* $O(n)$
- *has energy cost* $O(\log\log n)$ *with probability at least* $1 - \frac{1}{n}$.

Inefficient solution

Basic Algorithm

`for` $k = 1, 2, \ldots$ *run phase* k:

`repeat` $d \cdot k$ `times` *a trial:*

each station sends with probability $1/2^k$ and listens and counts the number of successful steps

`if` the number of successful steps is close to $d \cdot k / e$ `then`

$n_0 \leftarrow 2^k$, `halt`

Energy cost of Basic Algorithm

- each station listens all the time
- $O(\log n)$ trials necessary in order to get high probability for the right size of k

Improvements idea I

- only stations that send listen
- counting the number of successes by the successful senders only – efficient algorithms with $O(\log\log n)$ energy cost JKZ, PODC'2002
- results of counting acknowledged for each size of k
- $\Rightarrow$ we get a problem!
  $\Theta(\log n)$ times the size of k changed

Basic Algorithm

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Improvements ideas II

- the results acknowledged for all stations not for every k but only those that have the form 2^j
- the number of phases executed may double, but energy cost due to broadcasting reduced to $O(\log\log n)$

Improvement III

Problem: a station may participate in many "Ethernet trials", hard to guarantee a bound on energy cost

Solution: every station participates in an "Ethernet trial" at most once

Features of the algorithm

- a slight modification of Basic Algorithm
- extremely simple
- intuitively correct
- where is the problem?

Original algorithm

- trials independent stochastically,
- expected number of successes given by theory of Bernoulli trials
- the deviations from the expected value - estimated by Chernoff Bounds

Problems with analysis of modified algorithm

1. easy: showing that whp the number of stations already used for Ethernet trial is small
2. intuition: small number of stations eliminated in this way should not change probability properties very much
3. is this true? strong stochastic dependences,
4. theory of Bernoulli trials does not apply

Technical contribution

- correctness analysis of the algorithm despite stochastic dependencies
- a new paradigm for constructing energy efficient solutions with provably low complexity

Main phenomenon

For a k such that $k \leq \log n - 6$ number k is accepted with probability $O(\frac{1}{n^2})$.

For $k = \lceil \log n \rceil$, number k is not accepted with probability $O(\frac{1}{n^2})$.

Corollary If k_0 is the first k accepted by the algorithm, then 2^k is a good approximation of the network size.

Success probabilities

Let the number of unused stations before a step be $\geq n/2$.

Then for $k = \lceil \log n \rceil$, the probability that exactly one station sends is at least 0.1 .

Then for $k \leq \log n - 6$ and some constant c', the probability that exactly one station sends is $\leq p_k = \frac{n}{2^{k-1}} \left(1 - \frac{1}{2^k}\right)^{n/2}$.
(very small!)

Relating independent and dependent trials

consider a value of k such that $k \leq \log n - 6$

- $x_1, x_2, \ldots$ - independent binary random variables with probability p_k of choosing 1
- $w_1, w_2, \ldots$ - indicator variables for success in consecutive trials for this k

Sum of variables x_i easy to estimate (Bernoulli trials), for variables w_i it is difficult!

Main Lemma

Let $M = 2^{n/2}$, $k \leq \log n - 6$. For each $c > 0$ holds:

$$\mathbf{P}\left[\sum_{i=1}^{dk} w_i > c \,|\, \mathcal{S}_k^1\right] \leq \mathbf{P}\left[\sum_{i=1}^{dk} x_i > c\right] + \frac{2^{dk}-1}{M \cdot \mathbf{P}\left[\mathcal{S}_k^1\right]}$$

where $\mathcal{S}_k^j$ is the event that there are at least $n/2$ unused stations at jth trial for a given value of k.

Recall that $\mathbf{P}\left[\mathcal{S}_k^1\right] \approx 1$.

Final remarks:

- IEEE Standard 802.11
- after some modifications the algorithm runs in a system where a station sending cannot monitor the communication channel
- JKZ, Euro-Par 2002