Self-stabilizing population of mobile agents

Z. Gołębiewski
University of Wroclaw

M. Kutyłowski, F. Zagórski
Wrocław University of Technology

T. Łuczak
Adam Mickiewicz University, Poznań

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We consider a network:

- consisting of $n$ nodes
- fully connected:
  a node can send a message directly to another node

.. and mobile agents in such a network.
mobile agent is a unit that can migrate through the system.

activities of an agent:

1. can migrate to an arbitrary chosen node,
2. can reproduce itself, i.e. generate its copies at the node where it resides,
3. can kill other agents or become killed (e.g. by another agent or the system)
Time is divided into synchronous rounds.

Each round consists of 2 phases:

- **move phase**: an agent can migrate to another node,
- **evolution phase**: an agent can
  1. reproduce or become killed,
  2. perform internal tasks
Application of agent systems

**worms** an agent is a worm that tries to infect as many nodes as possible:

- it tries to replicate through the system (but no more than one worm in a node)
- it tries to behave so that it is hard to catch all copies of a worm
Application of agent systems

monitoring agents the agents perform some supervision and protect a system consisting of many PCs, they:

▶ should survive in the system even if a substantial number of PCs is taken over by the adversary
▶ should be hard to remove even if a (malicious) administrator wants them to switch off for a moment
Main goals

Design an algorithm that

- keeps number of agents in the system around pre-defined level $\alpha = \alpha(n)$,
- agents cannot leave any information in nodes,
- agents can communicate only with the agents residing in the same node.
Previous work


Similar algorithms of controlling agents population, but:
- agents leave traces at host nodes
- no analytic results, only simulations
Our Algorithm

Algorithm executed by an agent:

**Move:** Pick a node uniformly at random; move to this node (agents’ choices are independent)
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Algorithm executed by an agent:

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**Evolution:**

- if (there is exactly one agent in the node)
  - then with probability $p$ it creates a new agent in this node,
  - else fight!

exactly one of the agents survives (other agents in this node are killed).
There are two opposite mechanisms integrated in the protocol:

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  (agents replicate, killing occurs rarely since they do not meet frequently),
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- if the number of agents is low, then the number of agents is increasing
  (agents replicate, killing occurs rarely since they do not meet frequently),

- if the number of agents is high, then the number of agents is reduced
  (agents meet frequently, killings outnumber replications)
The analysis is based on the labeled combinatorial structures and their exponential multivariate generating functions (EMGF). They allow us to compute easily:

- **the expected number** of the number of born and killed agents in a network, (possible with the ” approach)
- **the variance** of the number of born and killed agents in a network - handling with dependencies between agents!
Basic definitions and notation

Let \( Z \) be the atomic class, i.e. \( Z = \{ 1 \} \), and \( 1 \) be a labeled atom of size 1 (an atom corresponds to the agent).
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- The EGF of the atomic class $\mathcal{Z}$ is $Z(z) = z$. 
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The EGF of the class $\mathcal{P}$ satisfies

$$P(z) = \sum_k P_k(z) = \sum_k \frac{1}{k!} (Z(z))^k = \sum_k \frac{z^k}{k!} = e^z.$$
Basic facts

The key fact concerning multivariate generating functions is that the moment of order 1 of a parameter $\chi_1$ is given by the formula

$$E_{Q(n)}[\chi_1] = \frac{[z^k] \partial_{u_1} P^{(n)}(z, 1, 1)}{[z^k] P^{(n)}(z, 1, 1)}$$

(1)

where $[z^k] S(z)$ extracts the coefficient of $z^k$ in the power series $S(z)$, and $\partial_{u_1} := \frac{\partial}{\partial u_1}$. 
Similarly, the moment of order 2 of a parameter $\chi_1$ equals

$$E_{P^{(n)}}[\chi_1^2] = \left[ z^k \partial_{u_1} \right] \frac{P^{(n)}(z, 1, 1)}{[z^k] P^{(n)}(z, 1, 1)} + \left[ z^k \partial_{u_1} \right] \frac{P^{(n)}(z, 1, 1)}{[z^k] P^{(n)}(z, 1, 1)}$$

(2)

where $\partial_{u_1}^2 := \frac{\partial^2}{\partial u_1^2}$. 
Step 1

- The class \( P^{(n)} \) that describes all nodes in a network is defined as a product of \( n \) classes \( P \).
- The EGF of the class \( P^{(n)} \) is given by formula
\[
P^{(n)}(z) = (P(z))^n.
\]
Step 2

We define parameters $\chi_1$ and $\chi_2$.

- $\chi_1$ associates the number of nodes with exactly one agent to an arrangement of agents in the nodes,

- Since $\frac{z^1}{1!} = z$ describes the nodes with exactly one agent, we will multiply it by a formal variable $u_1$ that marks $\chi_1$. 
Step 2

- $\chi_2$ associates the number of killed agents to an arrangement of agents in the nodes.
- Since $\frac{z^k}{k!}$ describes the node with exactly $k$ agents we will multiply it by a formal variable $u_2^{k-1}$ that marks $\chi_2$.
- $\sum_{k \geq 2} \frac{u_2^{k-1} z^k}{k!}$ describes the nodes with more than one agent.
Therefore we get the following exponential multivariate generating function (EMGF)

\[ P^{(n)}(z, u_1, u_2) = \left( 1 + u_1 z + \left( \sum_{k \geq 2} \frac{u_2^{k-1} z^k}{k!} \right) \right)^n = \ldots \]

\[ = \left( 1 + u_1 z + \frac{1}{u_2} (e^{u_2 z} - u_2 z - 1) \right)^n. \]
Lemma

Let \( k \) be the number of agents in a network at the beginning of a round. Then,

\[
E[\chi_1] = k \left( 1 - \frac{1}{n} \right)^{k-1}
\]

\[
\text{Var}[\chi_1] = k \left( \left( 1 - \frac{1}{n} \right)^{k-1} - k \left( 1 - \frac{1}{n} \right)^{2(k-1)} \right) + \frac{k(k-1)(n-1)(n-2)^{k-2}}{n^{k-1}}.
\]
Corollary

Let \( B \) denote the number of agents born in a round such that there are \( k \) agents in the network immediately before the round. Then

\[
E[B] = p \cdot k \left(1 - \frac{1}{n}\right)^{k-1}
\]

\[
Var[B] = pk \left(1 - \frac{1}{n}\right)^{k-1} - p^2 k^2 \left(1 - \frac{1}{n}\right)^{2k-2}
\]

\[
+ p^2 k(k-1) \left(1 - \frac{1}{n}\right) \left(1 - \frac{2}{n}\right)^{k-2}.
\]
The expected number and variance of born agents, for \( n = 1000 \), \( p = 0.25 \)
Lemma

\( n = \text{the number of nodes in a network,} \)
\( p = \text{the agents reproduction probability,} \)
\( k = \text{the number of agents in a network at the beginning of a round,} \)
\( K = \text{the number of agents killed in a network after the round.} \)

Then

\[
E[K] = k - n + n \left( 1 - \frac{1}{n} \right)^k \tag{7}
\]

\[
\text{Var}[K] = n(n-1) \left( 1 - \frac{2}{n} \right)^k + n \left( 1 - \frac{1}{n} \right)^k - n^2 \left( 1 - \frac{1}{n} \right)^{2k}.
\]
The expected number and variance of killed agents, for $n = 1000$, $p = 0.25$
**Equilibrium condition**

**Definition**

We say that the process considered is in the **Equilibrium Point**, when the expected change of the number of agents in a round equals 0,

i.e. the expected number of agents born in a round is equal to the expected number of agents killed in a round: $E[B] = E[K]$. 
Equilibrium condition

**Theorem**

Let $n =$ the number of nodes in a network, $p =$ the reproduction probability, $k =$ the number of agents at the beginning of a round. Then the Equilibrium Point

$$pk \left(1 - \frac{1}{n}\right)^{k-1} = k - n + n \left(1 - \frac{1}{n}\right)^k$$

is reached for

$$k \approx \frac{2p}{1 + p} n.$$
Corollary

The number of agents at the Equilibrium Point can be established on any chosen value $\alpha \cdot n$, $0 < \alpha < 1$ by choosing $p \approx \frac{\alpha}{2-\alpha}$.
### Experimental results

<table>
<thead>
<tr>
<th>$n = 1000$</th>
<th>according to (8)</th>
<th>average number in simulations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p = 0.1$</td>
<td>178.46</td>
<td>180</td>
</tr>
<tr>
<td>$p = 0.25$</td>
<td>386.054</td>
<td>385</td>
</tr>
<tr>
<td>$p = 1$</td>
<td>1000.58</td>
<td>1000</td>
</tr>
</tbody>
</table>

Equilibrium Point versus the average number of agents for $n = 1000$
Equilibrium Point for different reproduction probabilities, for $n = 1000$
Evolution of the number of agents, an example simulation for $n = 1000$
Final Remarks

- The rate of convergence to the Equilibrium Point has not been covered by the paper,
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- the rate of convergence to the Equilibrium Point has not been covered by the paper,
- computed values of the variances of variables $B$ and $K$ are quite low—this influences on high rate of convergence.
- Numerical experiments show that the convergence is fast regardless of the initial situation!
Thanks for your attention