Security of Okamoto Identification Scheme - a Defense against Ephemeral Key Leakage and Setup

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Identification

Identification Scheme (IS)

- **prover** – proves his identity,
- **verifier** – accepts or rejects the proof

Attributes of the authenticator

- what the prover **has** (key, token, etc.),
- what the prover **knows** (secret key, password, etc.),
- what the prover **are** (e.g. biometric)

We concentrate on “**what the authenticator knows**” methodology.
Some known schemes

Some dedicated construction

- **Schnorr, C.P.:**
  Efficient signature generation by smart cards.

- **Fiat, A., Shamir, A.:**
  How To Prove Yourself: Practical Solutions to Identification and Signature Problems.

- **Feige, U., Fiat, A., Shamir, A.:**
  Zero-knowledge proofs of identity.

- **Guillou, L.C., Quisquater, J.J.:**
  A practical zero-knowledge protocol fitted to security microprocessor minimizing both transmission and memory.

- **Okamoto, T.:**
  Provably Secure and Practical Identification Schemes and Corresponding Signature Schemes.

- **Kurosawa, K., Heng, S.H.:**
  Identity-Based Identification Without Random Oracles

- **Canetti, R., Goldreich, O., Goldwasser, S., Micali, S.:**
  Resettable zero-knowledge (extended abstract).

- **Bellare, M., Fischlin, M., Goldwasser, S., Micali, S.:**
  Identification Protocols Secure against Reset Attacks.
Asymmetric cryptography setup
- the prover has a long term secret key
- the verifier has the corresponding public key

Zero Knowledge Proof
- the verifier is convinced,
- gets no information about the prover’s secret.
Three rounds

- **commitment** the prover sends a commitment to some random ephemeral value.
- **challenge** the verifier random unpredictable challenge.
- **response** the prover sends the result of some computations over the challenge, the secret and the ephemeral value.

Verification

The prover is accepted if the response ”agrees” with the computation involving the commitment, the challenge, the response and the public key of the prover.
General Construction

Protocol

\( x \) for commitment
\( c \) for challenge
\( s \) for proof
Deniability

**Deniable Identification**

**Simulatability**: The prover without the secret key can produce the transcript itself.

**Distinguisher**

Cannot tell

- whether the transcript was a result of the regular protocol execution.
- or the transcript was simulated.

even if it was given the secret key.
Okamoto identification scheme

Initialization Stage

params $\leftarrow \text{ParGen}(1^\lambda)$: Let $G = (p, q, g, G) \leftarrow g(1^\lambda)$, s.t. DL assumption holds. Set params $= (p, q, g_1, g_2, G)$.

KeyGen(): $\text{sk} = a_1, a_2 \leftarrow \mathbb{Z}_q^*$, $\text{pk} = A = g_1^{a_1}g_2^{a_2}$. Output $(\text{sk}, \text{pk})$.

Figure: The Okamoto identification scheme.
Okamoto identification scheme

Operation Stage

\[ \pi(\mathcal{P}(a_1, a_2), \mathcal{V}(A)) : \]

1. **\( \mathcal{P} : \)** \( x_1, x_2 \in R \mathbb{Z}_q^*, \) \( X = g_1^{x_1} g_2^{x_2} \)
   sends \( X \) to the verifier \( \mathcal{V} \).

2. **\( \mathcal{V} : \)** \( c \in R \mathbb{Z}_q^* \),
   sends \( c \) to the prover \( \mathcal{P} \).

3. **\( \mathcal{P} : \)** \( s_1 = x_1 + a_1 c \) \( s_2 = x_2 + a_2 c \)
   sends \( s_1, s_2 \) to the verifier \( \mathcal{V} \).

Verifier accepts the Prover iff

\[ g_1^{s_1} g_2^{s_2} \equiv X A^c \]
## Okamoto identification scheme

### Operation Stage

<table>
<thead>
<tr>
<th>$\mathcal{P}(a_1, a_1)$</th>
<th>$\mathcal{V}(A = g_1^{a_1} g_2^{a_2})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1, x_2 \in_R \mathbb{Z}_q^*$</td>
<td>$X = g_1^{x_1} g_2^{x_2}$</td>
</tr>
<tr>
<td>$s_1 = x_1 + a_1 c$</td>
<td>$s_2 = x_2 + a_2 c$</td>
</tr>
</tbody>
</table>

Accept iff $g_1^{s_1} g_2^{s_2} = X A^c$
Okamoto IS vs. Ephemeral Leakage

Krzywiecki, Kutyłowski

Okamoto identification scheme
Deniability

Protocol Simulation

1. Simulator chooses $\tilde{s}_1, \tilde{s}_2, \tilde{c}$ first
2. Then $\tilde{X} = (g_1^{\tilde{s}_1} g_2^{\tilde{s}_2} / A^{\tilde{c}})$.

The tuples
$T = (X, c, s_1, s_2)$ - from the protocol execution
$\tilde{T} = (\tilde{X}, \tilde{c}, \tilde{s}_1, \tilde{s}_2)$ - simulated
are identically distributed.
Device based authentication

Device
Small hardware which *securely* store the authentication keys inside (e.g. smartcards).

Adversaries Attacks
- tries to *extract* what was *put* inside,
- tries to *manipulate* what is *inside*,
- ...

Common threats:
- invasive attack,
- power analysis,
- emission of radiation,
- ...

Okamoto identification scheme

Deniability

Device architecture

HSM: Secret keys

Output

Program... Random Numbers...

Input
Okamoto IS is not secure if $\bar{x}$ is known to the adversary. $A$ can easily compute the secret key $a_i = (s_i - \bar{x}_i)/c$. 
Security experiment

The experiment $\text{Exp}_{\text{CPE}, \lambda, \ell}^\text{CPE, IS}$:

- **Init stage**: params $\leftarrow \text{ParGen}(1^\lambda)$, $(sk, pk) \leftarrow \text{KeyGen}()$. $\mathcal{A} = (\tilde{\mathcal{P}}, \tilde{\mathcal{V}})$ given the public key pk.

- **Query stage**: $\mathcal{A}$ runs a polynomial number $\ell$ of $\pi(\mathcal{P}^{\tilde{x}_i}(sk, pk), \tilde{\mathcal{V}}(pk, \tilde{x}_i)$ collecting view $v^{\mathcal{P}, \tilde{\mathcal{V}}, \tilde{x}(\ell)}$, where $\tilde{x}_i \in \{\tilde{x}_1, \ldots, \tilde{x}_\ell\}$ are injected.

- **Impersonation stage**: $\mathcal{A}$ runs the protocol $\pi(\tilde{\mathcal{P}}(pk, v^{\mathcal{P}, \tilde{\mathcal{V}}, \tilde{x}(\ell)}), \mathcal{V}(pk))$
Adversary advantage

The advantage of $\mathcal{A}$ in the experiment $\text{Exp}_{\text{IS}}^{\text{CPE},\lambda,\ell}$ as probability of acceptance in the impersonation stage:

$$\text{Adv}(\mathcal{A}, \text{Exp}_{\text{IS}}^{\text{CPE},\lambda,\ell}) = \Pr[\pi(\tilde{\mathcal{P}}(pk, v^P, \tilde{\nu}, \tilde{x}(\ell)), \nu(pk)) \rightarrow 1].$$

The identification scheme is secure if it is negligible in $\lambda$.

Security of identification scheme

$\mathcal{A}$ probability of acceptance is negligible in $\lambda$. 

Chosen Prover Ephemeral
Bilinear Map

Let $G_T$ be another group of a prime order $q$. We assume that $\hat{e} : G \times G \rightarrow G_T$ is a bilinear map s.t. following condition holds:

1) \textit{Bilinearity}: \forall a, b \in \mathbb{Z}_q^*, \forall g, g' \in G: \hat{e}(g^a, g'^b) = \hat{e}(g, g')^{ab}.

2) \textit{Non-degeneracy}: \hat{e}(g, g) \neq 1.

3) \textit{Computability}: $\hat{e}$ is efficiently computable.

New generator

Let $H : \{0, 1\}^* \rightarrow G$ be a hash function.

We compute another element of $G$ denoted by $\hat{g}$. 
Modified Okamoto identification scheme

**Operation Stage**

\[\mathcal{P}(a_1, a_2) \quad \mathcal{V}(A = g_1^{a_1} g_2^{a_2})\]

\[\begin{align*}
X_1, X_2 \in_R \mathbb{Z}_q^*, \\
X &= g_1^{x_1} g_2^{x_2} \\
S_1 &= \hat{g}^{x_1 + a_1 c} \\
S_2 &= \hat{g}^{x_2 + a_2 c} \\
\hat{g} &= \mathcal{H}(X|c) \\
\hat{S}_1 &= S_1, S_2 \\
\hat{g} &= \mathcal{H}(X|c) \\
\hat{e}(S_1, g_1) \cdot \hat{e}(S_2, g_2) &= \hat{e}(\hat{g}, X \cdot A^c)
\end{align*}\]
Modified Okamoto identification scheme
Deniability

Protocol Simulation for Passive Adversary

1. Simulator chooses \( \tilde{s}_1, \tilde{s}_2, \tilde{c} \) first
2. \( \tilde{X} = (g_1^{\tilde{s}_1} g_2^{\tilde{s}_2} / A^{\tilde{c}}) \).
3. \( \hat{g} = H(\tilde{X}|\tilde{c}) \)
4. \( \tilde{S}_1 = \hat{g}^{\tilde{s}_1}, \tilde{S}_2 = \hat{g}^{\tilde{s}_2} \)

The tuples
\( T = (X, c, S_1, S_2) \) - from the protocol execution
\( \tilde{T} = (\tilde{X}, \tilde{c}, \tilde{S}_1, \tilde{S}_2) \) - simulated
are identically distributed.
CDH Breaking

1. given $\text{CDH}(g, g^\alpha, g^\beta)$
2. set $A = g^\alpha$
3. set $a_2, \omega \leftarrow_R \mathbb{Z}_q^*$
4. set $g_1 = g$, $g_2 = g^\omega$
5. we have $g_1^{a_1} = A/g_2^{a_2}$

We simulate Query stage in ROM. We use rewinding technique
Protocol Simulation for Active Adversary

1. **ROM table** $O_H$: Three columns $I, H, r$: for the input, the output and the masked exponent respectively.

2. **New query**: $r_i \leftarrow_R \mathbb{Z}^*_q$, compute $H_i = g^{r_i}$, insert $(I_i, H_i, r_i)$, return $H_i$.

   **Commitment**: When injected ephemeral $\bar{x}_1, \bar{x}_2$, compute $\tilde{X} = g_1^{\bar{x}_1} g_2^{\bar{x}_2}$ and send $\tilde{X}$ to the verifier

   **Proof**: On receiving $\tilde{c}$, call $O_H(\bar{X}|\tilde{c})$, locate and retrieve the corresponding $g^r$ and $r$. We set $\hat{g} = g^r$.

   Compute:
   
   $\tilde{S}_1 = (g_1^{x_1})^r (A/g_2^{a_2})^{rc} = \hat{g}^{\bar{x}_1+a_1c}$
   
   $\tilde{S}_2 = (g_2^{x_2})^r (g_2^{a_2})^{rc} = \hat{g}^{\bar{x}_2+a_2c}$

Verification holds. $T = (X, c, S_1, S_2)$, and $\tilde{T} = (\tilde{X}, \tilde{c}, \tilde{S}_1, \tilde{S}_2)$ identically distributed.
Then we use the *rewinding technique*:

1. we run protocol twice for
2. the same fixed commitment $X$,
3. use different challenges $c, c'$
4. in ROM inject $g^\beta$
5. get responses $S_1, S_2$, and $S'_1, S'_2$.
6. two resulting tuples $(X, c, S_1, S_2), (X, c', S'_1, S'_2)$
7. these enable us to break the underlying GDH$(g, g^\alpha, g^\beta)$. 
Security rationale

Problems for the Adversary

1. From $\hat{g}^{\bar{x} + ac}$ it is hard to get $a$

2. If you know $\bar{x}, c$ you can compute $\hat{g}^a$

3. Knowing $\hat{g}_1^{a_1}, \ldots, \hat{g}_\ell^{a_1}$
   still it is hard to compute $\hat{g}_n^{a_1}$
   for completely new element $\hat{g}_n$

4. Knowing $\hat{g}_1^{a_2}, \ldots, \hat{g}_\ell^{a_2}$
   still it is hard to compute $\hat{g}_n^{a_2}$
   for completely new element $\hat{g}_n$
Shifting computations to Cloud
Possible Advantages

"Gray" Secure Module
1. user retain "Gray" Secure Module
2. "Gray" Secure Module – black box

"Yellow" Insecure Module
1. yellow part can be outsourced to cloud
2. yellow part – white box

Adversary cloud cannot:
- extract long term secret keys,
- impersonate user
Thank You